

Higher Spin Currents in the Enhanced $\mathcal{N} = 3$ Kazama-Suzuki Model

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Abstract

The $\mathcal{N} = 3$ Kazama-Suzuki model at the ‘critical’ level has been found by Creutzig, Hikida and Ronne. We construct the lowest higher spin currents of spins $(\frac{3}{2}, 2, 2, 2, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, 3)$ in terms of various fermions. In order to obtain the operator product expansions (OPEs) between these higher spin currents, we describe three $\mathcal{N} = 2$ OPEs between the two $\mathcal{N} = 2$ higher spin currents denoted by $(\frac{3}{2}, 2, 2, \frac{5}{2})$ and $(2, \frac{5}{2}, \frac{5}{2}, 3)$ (corresponding 36 OPEs in the component approach). Using the various Jacobi identities, the coefficient functions appearing on the right hand side of these $\mathcal{N} = 2$ OPEs are determined in terms of central charge completely. Then we describe them as one single $\mathcal{N} = 3$ OPE in the $\mathcal{N} = 3$ superspace. The right hand side of this $\mathcal{N} = 3$ OPE contains the $SO(3)$ -singlet $\mathcal{N} = 3$ higher spin multiplet of spins $(2, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, 3, 3, 3, \frac{7}{2})$, the $SO(3)$ -singlet $\mathcal{N} = 3$ higher spin multiplet of spins $(\frac{5}{2}, 3, 3, 3, \frac{7}{2}, \frac{7}{2}, \frac{7}{2}, 4)$, and the $SO(3)$ -triplet $\mathcal{N} = 3$ higher spin multiplets where each multiplet has the spins $(3, \frac{7}{2}, \frac{7}{2}, \frac{7}{2}, 4, 4, 4, \frac{9}{2})$, in addition to $\mathcal{N} = 3$ superconformal family of the identity operator. Finally, by factoring out the spin- $\frac{1}{2}$ current of $\mathcal{N} = 3$ linear superconformal algebra generated by eight currents of spins $(\frac{1}{2}, 1, 1, 1, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 2)$, we obtain the extension of so-called $SO(3)$ nonlinear Knizhnik Bershadsky algebra.

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1 Introduction

One of the remarkable aspects of WZW models is that WZW primary fields are also Virasoro primary fields [1, 2]. The Virasoro zeromode acting on the primary state corresponding to the WZW primary field is proportional to the quadratic Casimir operator of the finite Lie algebra. Associating a conformal weight (or spin) to the primary state, we find the conformal weight (or spin) which is equal to the one half times the quadratic Casimir eigenvalues divided by the sum of the level and the dual Coxeter number of the finite Lie algebra [1, 2]. In particular, the adjoint representation at the ‘critical’ level (which is equal to the dual Coxeter number) has conformal weight $\frac{1}{2}$. For example, for $SU(N)$, the quadratic Casimir eigenvalue is given by $2N$ and the dual Coxeter number is N . Then we are left with the overall numerical factor $\frac{1}{2}$ which is the spin of adjoint fermion. For the diagonal coset theory [1], the conformal spin- $\frac{3}{2}$ current ($\mathcal{N} = 1$ supersymmetry generator) that commutes with the diagonal spin-1 current can be determined [3, 4, 5] and is given by the linear combination of two kinds of spin-1 currents and adjoint fermions. For $SU(2)$ case, this leads to the well known coset construction of the $\mathcal{N} = 1$ superconformal algebra [6]. For $SU(3)$ case, this leads to the coset construction of $\mathcal{N} = 1$ W_3 algebra [7, 8, 9, 10]. Moreover, for $SU(N)$, the $\mathcal{N} = 1$ higher spin multiplets are found in [11]. One can go one step further. By taking the adjoint spin- $\frac{1}{2}$ fermions in the second factor in the numerator of the diagonal coset model [1], the coset construction of the $\mathcal{N} = 2$ superconformal algebra is obtained [12] and the higher spin currents are observed and determined in [13, 14].

One can consider the above feature in the different coset model. There exists other type of coset model, so-called Kazama-Suzuki model [15, 16] which is described as

$$\frac{\hat{S}U(N+M)_k \oplus \hat{S}O(2NM)_1}{\hat{S}U(N)_{k+M} \oplus \hat{S}U(M)_{k+N} \oplus \hat{U}(1)_{NM(N+M)(k+N+M)}}, \quad c = \frac{3kNM}{(k+N+M)}. \quad (1.1) \{?\}$$

The $\mathcal{N} = 2$ superconformal algebra, which is generated by the $\mathcal{N} = 2$ multiplet of spins $(1, \frac{3}{2}, \frac{3}{2}, 2)$, is realized for arbitrary level k in the coset model (1.1). When $M = 2$, the above coset is similar to the Wolf space coset [17] where the $\hat{S}U(M=2)_{k+N}$ factor in the

denominator of (1.1) is not present. At the “critical” level where the level k of spin-1 current in the $SU(N+M)$ factor in the numerator (1.1) is equal to the dual Coxeter number $(N+M)$,

$$k = N + M, \quad c = \frac{3}{2}NM, \quad (1.2) \{?\}$$

the above coset model (1.1) has $\mathcal{N} = 3$ supersymmetry [18, 19, 20]. According to the previous analysis, there exist $(N+M)^2 - 1$ adjoint spin- $\frac{1}{2}$ fermions residing in the $SU(N+M)$ factor of the numerator. Under the decomposition of $SU(N+M)$ into the $SU(N) \times SU(M)$, the adjoint representation of $SU(N+M)$ breaks into as follows: $(\mathbf{N} + \mathbf{M})^2 - 1 \rightarrow (\mathbf{N}^2 - 1, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{M}^2 - 1) \oplus (\mathbf{1}, \mathbf{1}) \oplus (\mathbf{N}, \overline{\mathbf{M}}) \oplus (\overline{\mathbf{N}}, \mathbf{M})$. The adjoint fermion in the trivial representation $(\mathbf{1}, \mathbf{1})$ of both $SU(N)$ and $SU(M)$ plays the important role in the construction of the ‘extra’ currents consisting of the extra $\mathcal{N} = 2$ multiplet of spins $(\frac{1}{2}, 1, 1, \frac{3}{2})$ besides the above $\mathcal{N} = 2$ current of spins $(1, \frac{3}{2}, \frac{3}{2}, 2)$ [21, 22, 23, 24, 25, 26].

One way to obtain these ‘extra’ currents is as follows. Starting with the above adjoint fermion corresponding to $(\mathbf{1}, \mathbf{1})$, one can calculate the OPEs between this single fermion (which is the lowest spin- $\frac{1}{2}$ current in the above $\mathcal{N} = 2$ multiplet) and the spin- $\frac{3}{2}$ currents of $\mathcal{N} = 2$ supersymmetry assuming that the above $\mathcal{N} = 2$ multiplet of spins $(1, \frac{3}{2}, \frac{3}{2}, 2)$ is known explicitly in the coset model (1.1) together with (1.2). Via the operator product expansions (OPEs) of $\mathcal{N} = 3$ superconformal algebra, one can determine the above two spin-1 currents residing in the $\mathcal{N} = 2$ multiplet $(\frac{1}{2}, 1, 1, \frac{3}{2})$. In other words, the first-order poles of the above OPE between the adjoint single fermion and the two spin- $\frac{3}{2}$ currents provide the two spin-1 currents respectively. Furthermore, by taking one of the spin-1 currents and one of the spin- $\frac{3}{2}$ currents of $\mathcal{N} = 2$ superconformal algebra and calculating the OPE between them (their $SO(3)$ indices are different from each other), one obtains the first-order pole which provides the spin- $\frac{3}{2}$ current residing in the above $\mathcal{N} = 2$ multiplet $(\frac{1}{2}, 1, 1, \frac{3}{2})$. Therefore, the $\mathcal{N} = 3$ superconformal algebra generated by the currents of spins $(\frac{1}{2}, 1, 1, 1, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 2)$ by adding the two $\mathcal{N} = 2$ multiplets $(1, \frac{3}{2}, \frac{3}{2}, 2)$ and $(\frac{1}{2}, 1, 1, \frac{3}{2})$ and computing the OPEs between them is realized in the enhanced $\mathcal{N} = 3$ Kazama-Suzuki model (1.1) together with (1.2) [18].

In this paper, we study the higher spin currents in the coset model (1.1). For the $\mathcal{N} = 3$ holography [18], the deformation breaks the higher spin symmetry and induces the mass to the higher spin fields [19, 20]. The masses are not generated for the $SO(3)_R$ singlet higher spin fields at the leading order of $\frac{1}{c}$ while the mass formula for the $SO(3)_R$ triplet higher spin fields looks like the Regge trajectory on the flat spacetime. So far it is not known what is the higher spin symmetry algebra for the higher spin currents together with $\mathcal{N} = 3$ superconformal algebra. It would be interesting to see the higher spin symmetry algebra between the low higher spin currents explicitly. For finite (N, M) (or finite c) in the coset model, we would like

to observe the marginal operator which breaks the higher spin symmetry keeping the $\mathcal{N} = 3$ supersymmetry. Furthermore, we should obtain the explicit higher spin symmetry algebra, where the structure constants on the right hand side of OPEs depend on (N, M) explicitly, in order to calculate the mass formula as in the large c limit [19, 20].

We expect that the $\mathcal{N} = 3$ lowest higher spin multiplet of spins $(\frac{3}{2}, 2, 2, 2, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, 3)$ can be obtained by adding the two $\mathcal{N} = 2$ higher spin multiplets $(\frac{3}{2}, 2, 2, \frac{5}{2})$ and $(2, \frac{5}{2}, \frac{5}{2}, 3)$ (or by adding the spin one to the $\mathcal{N} = 3$ multiplet of $\mathcal{N} = 3$ superconformal algebra we have described above). Once we know the higher spin currents explicitly, then we can perform the OPEs between them and obtain the higher spin algebra for low higher spin currents. Then how we can obtain the higher spin currents explicitly besides the $\mathcal{N} = 3$ currents generated by $\mathcal{N} = 3$ superconformal algebra? As in the first paragraph, we return to the construction of WZW currents in the coset model (1.1). One of the usefulness of this construction is that we can obtain the higher spin currents directly and due to the $\mathcal{N} = 3$ supersymmetry, we can determine the other higher spin currents after the lowest higher spin current is fixed in the given $\mathcal{N} = 3$ multiplet. For example, once the higher spin- $\frac{3}{2}$ current is determined completely, then the higher spin-2, $\frac{5}{2}$ and 3 currents can be obtained with the help of spin- $\frac{3}{2}$ currents of $\mathcal{N} = 3$ superconformal algebra.

Furthermore, the general feature in the OPE between any two quasiprimary currents is used [1]. Because the left hand side of any OPE can be calculated from the WZW currents explicitly, the pole structures of the OPEs are known. From these, we should express them in terms of the known $\mathcal{N} = 3$ currents and the known higher spin currents by assuming that the right hand sides of the OPEs contain any multiple products between them. If we cannot describe the poles of the OPEs in terms of the known (higher spin) currents, then we should make sure that the extra terms should transform as a new (quasi)primary current. This will consist of the component of next higher spin multiplet. When the poles of the OPEs are described by low spin, then it is easy to figure out the right candidate for the composite currents at each pole of the OPEs. As the spins of the left hand side of the OPEs increase, then it is not easy to write down all possible terms correctly. We use the $SO(3)$ index structure in the (higher spin) currents and according to the $SO(3)$ index structure of the left hand side of the OPEs, the right hand side of the OPEs should preserve the $SO(3)$ invariance. In other words, if the left hand side of the OPEs transforms as a singlet, then the right hand side of the OPEs should be a singlet under the $SO(3)$. Similarly, the $SO(3)$ vector (free) index can arise both sides of the OPEs and we will see the appearance of the new higher spin currents with $SO(3)$ vector index.

We would like to construct the complete 36 OPEs between the above eight higher spin

currents for generic central charge. Now we can proceed to the $\mathcal{N} = 2$ superspace from these component results and all the expressions are given for $(N, M) = (2, 2)$. Of course we can stay at the component approach but we should introduce more undetermined quantities we should determine. Let us replace the structure constants with arbitrary coefficients. Then we have the complete OPEs in the $\mathcal{N} = 2$ superspace with undetermined structure constants. We use the Jacobi identities to fix the structure constants. In general, the new $\mathcal{N} = 2$ primary higher spin current transforming as a primary current under the $\mathcal{N} = 2$ stress energy tensor can appear on the right-hand side of the OPEs as the spins of the currents increase. The above 8 higher spin currents can be represented by two $\mathcal{N} = 2$ multiplets. Similarly, the 8 currents of the $\mathcal{N} = 3$ (linear) superconformal algebra can be combined into two $\mathcal{N} = 2$ multiplets (as described before). Then we can use the Jacobi identities by choosing one $\mathcal{N} = 2$ current and two $\mathcal{N} = 2$ higher spin currents. We cannot use the Jacobi identities by taking three $\mathcal{N} = 2$ higher spin currents because, if we consider the OPE between any $\mathcal{N} = 2$ higher spin currents and another new $\mathcal{N} = 2$ higher spin current, we do not know this OPE at this level. Therefore, the three quantities used for the Jacobi identities are given by one $\mathcal{N} = 2$ current and two $\mathcal{N} = 2$ higher spin currents. We can also consider the combination of one $\mathcal{N} = 2$ higher spin current and two $\mathcal{N} = 2$ currents, but this will do not produce any nontrivial equations for the unknown coefficients. They are satisfied trivially.

After we obtain the complete three $\mathcal{N} = 2$ OPEs, then it is straightforward to write down them as a single $\mathcal{N} = 3$ OPE (or as the component results). We observe that on the right hand side of the $\mathcal{N} = 3$ OPE, there exist three types of $\mathcal{N} = 3$ higher spin multiplets. There are two $SO(3)$ singlets of spins $(2, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, 3, 3, 3, \frac{7}{2})$ and $(\frac{5}{2}, 3, 3, 3, \frac{7}{2}, \frac{7}{2}, \frac{7}{2}, 4)$ and one $SO(3)$ triplet where each multiplet has the spins $(3, \frac{7}{2}, \frac{7}{2}, \frac{7}{2}, 4, 4, 4, \frac{9}{2})$. The presence of $SO(3)$ triplet higher spin multiplet is crucial to the $\mathcal{N} = 3$ OPE. We observe that the $SO(3)$ vector index for the last $\mathcal{N} = 3$ higher spin multiplet is contracted with the one appearing in the fermionic coordinates of $\mathcal{N} = 3$ superspace. In other words, one can use the $SO(3)$ invariant tensor of rank 3 and make a contraction with both two fermionic coordinates and the above $\mathcal{N} = 3$ $SO(3)$ triplet higher spin current. Because the spin of the first-order pole with two fermionic coordinates is given by zero, the sum of the two spin $\frac{3}{2}$ of the left hand side should appear on the right hand side. Note that the above lowest higher spin multiplet of spins $(\frac{3}{2}, 2, 2, 2, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, 3)$ is a $SO(3)$ singlet.

In section 2, we review the $\mathcal{N} = 3$ stress energy tensor, the primary higher spin multiplets and the realization of the $\mathcal{N} = 3$ superconformal algebra in the above coset model.

In section 3, we construct the lowest eight higher spin currents for generic central charge c explicitly.

In section 4, we obtain the fundamental OPEs between the higher spin currents found in previous section for $(N, M) = (2, 2)$ case.

In section 5, we present how we can determine the next higher spin currents.

In section 6, we calculate the Jacobi identities for the three $\mathcal{N} = 2$ OPEs and determine the structure constants completely.

In section 7, based on the previous section, the component result can be obtained. Furthermore, we describe its $\mathcal{N} = 3$ OPE.

In section 8, by factoring out the spin- $\frac{1}{2}$ current of $\mathcal{N} = 3$ superconformal algebra, we obtain the (minimally) extended Knizhnik Bershadsky algebra [27, 28].

In Appendices A, B, \dots, H , we describe some details which are necessary to the previous sections.

The packages [29, 30] are used all the times.

Some of the relevant works in the context of [15, 16] are given in [31]-[51].

2 The eight currents of $\mathcal{N} = 3$ superconformal algebra in the coset model: Review

In this section, we describe the 8 currents of the $\mathcal{N} = 3$ (linear) superconformal algebra in the $\mathcal{N} = 3$ superspace, where $SO(3)$ symmetry is manifest. Then the corresponding $\mathcal{N} = 3$ superconformal algebra, which consists of 9 nontrivial OPEs in the component approach (in Appendix A), can be expressed in terms of a single $\mathcal{N} = 3$ (super) OPE. We describe the $\mathcal{N} = 3$ (super) primary higher spin current, in an $SO(3)$ symmetric way, under the $\mathcal{N} = 3$ stress energy tensor. The $\mathcal{N} = 3$ higher spin current, in general, transforms as a nontrivial representation under group $SO(3)$. Furthermore, the superspin is, in general, given by the positive integer or half integer Δ , but its lowest value $\Delta = \frac{3}{2}$ will be considered later when the OPEs between them are calculated for generic central charge. The OPEs between the 8 currents and the 8 higher spin currents in the component approach are also given (in Appendix B).

2.1 The $\mathcal{N} = 3$ stress energy tensor

The $\mathcal{N} = 3$ stress energy tensor can be described as [23]

$$\begin{aligned} \mathbf{J}(Z) &= \frac{1}{2}i\Psi(z) + \theta^i \frac{i}{2}J^i(z) + \theta^{3-i} \frac{1}{2}G^i(z) + \theta^{3-0} T(z) \\ &= \frac{1}{2}i\Psi(z) + \theta^1 \frac{i}{2}J^1(z) + \theta^2 \frac{i}{2}J^2(z) + \theta^3 \frac{i}{2}J^3(z) \end{aligned}$$

$$\begin{aligned}
& + \theta^1 \theta^2 \frac{1}{2} G^3(z) + \theta^2 \theta^3 \frac{1}{2} G^1(z) + \theta^3 \theta^1 \frac{1}{2} G^2(z) + \theta^1 \theta^2 \theta^3 T(z) \\
& \equiv \left(\frac{i}{2} \Psi, \quad \frac{i}{2} J^i, \quad \frac{1}{2} G^i, \quad T \right).
\end{aligned} \tag{2.1} \{?\}$$

The $\mathcal{N} = 3$ superspace coordinates can be described as (Z, \overline{Z}) , where $Z = (z, \theta^i)$, $\overline{Z} = (\bar{z}, \bar{\theta}^i)$, and $i = 1, 2, 3$ and the index i is the $SO(3)$ -vector index. The left covariant spinor derivative is given by $D^i = \theta^i \frac{\partial}{\partial z} + \frac{\partial}{\partial \theta^i}$ and satisfies the anticommutators: $\{D^i, D^j\} = 2\delta^{ij} \frac{\partial}{\partial z}$ where the Kronecker delta δ^{ij} is the rank 2 $SO(3)$ symmetric invariant tensor. In the first line of (2.1), the summation over repeated indices (note that the $\mathcal{N} = 3$ stress energy tensor $\mathbf{J}(Z)$ is an $SO(3)$ -singlet) is taken.¹ The simplified notation θ^{3-0} is used for $\theta^1 \theta^2 \theta^3$. The complement $3 - i$ is defined such that $\theta^1 \theta^2 \theta^3 = \theta^{3-i} \theta^i$ (no sum over i). In the second line of (2.1), the complete 8 currents for the $\mathcal{N} = 3$ stress energy tensor are described in an expansion of Grassmann coordinates completely. The quartic- and higher-order terms in θ^i vanish owing to the property of θ^i . The 8 currents are given by a single spin- $\frac{1}{2}$ current $\Psi(z)$, three spin-1 currents $J^i(z)$ transforming as a vector representation under $SO(3)$, three spin- $\frac{3}{2}$ currents $G^i(z)$ transforming as a vector representation under $SO(3)$, and the spin-2 current $T(z)$. In particular, the spin- $\frac{1}{2}$ and spin-2 currents are $SO(3)$ singlets. The spin of θ^i is given by $-\frac{1}{2}$ (and the covariant spinor derivative D^i has spin $\frac{1}{2}$) and therefore the $\mathcal{N} = 3$ (super)spin of the stress energy tensor $\mathbf{J}(Z)$ is equal to $\frac{1}{2}$. Each term in (2.1) has a spin- $\frac{1}{2}$ value.

The $\mathcal{N} = 3$ OPE between the $\mathcal{N} = 3$ stress energy tensor and itself can be summarized by [21, 22, 23, 25]

$$\mathbf{J}(Z_1) \mathbf{J}(Z_2) = -\frac{1}{z_{12}} \frac{c}{12} + \frac{\theta_{12}^{3-0}}{z_{12}^2} \frac{1}{2} \mathbf{J}(Z_2) + \frac{\theta_{12}^{3-i}}{z_{12}} \frac{1}{2} D^i \mathbf{J}(Z_2) + \frac{\theta_{12}^{3-0}}{z_{12}} \partial \mathbf{J}(Z_2) + \dots, \tag{2.2} \{?\}$$

where summation over the repeated indices is assumed (the OPE between the $SO(3)$ -singlet current and itself), the fermionic coordinate difference for given index i is defined as $\theta_{12}^i = \theta_1^i - \theta_2^i$, and the bosonic coordinate difference is given by $z_{12} = z_1 - z_2 - \theta_1^i \theta_2^i$. Note that there exists $\mathbf{J}(Z_2)$ -term on the right-hand side of (2.2). Also the explicit component results (which will be described in next section) will be given in Appendix A.²

¹ We use boldface notation for the $\mathcal{N} = 3$ or $\mathcal{N} = 2$ multiplet to emphasize the fact that the corresponding multiplet has many component currents. For the $\mathcal{N} = 3$ multiplet, 8 independent component currents arise while, for the $\mathcal{N} = 2$ multiplet, 4 independent components arise.

² By assuming that there are four types of OPEs between the four component currents and the spin- $\frac{1}{2}$ current, $\Psi(z) \Psi(w)$, $J^i(z) \Psi(w)$, $G^i(z) \Psi(w)$, and $T(z) \Psi(w)$, we can also write down the $\mathcal{N} = 3$ OPE in (2.2). By simply taking $\theta_1^i = \theta_2^i = 0$ in the equation (2.2), we observe that the coefficient of the first term of (2.2) can be obtained from the OPE $\Psi(z) \Psi(w)$. The third term of the right hand side of (2.2) by acting the differential operator D_1^{3-i} and setting $\theta_1^i = \theta_2^i = 0$ can be obtained from the OPE $G^i(z) \Psi(w)$. Similarly, by acting $D_1^1 D_1^2 D_1^3$ on the equation (2.2) and putting $\theta_1^i = \theta_2^i = 0$, the singular terms can be obtained from the OPE $T(z) \Psi(w)$. The regularity of the OPE $J^i(z) \Psi(w)$ implies that there is no linear in θ_{12}^i in the equation (2.2).

2.2 The $\mathcal{N} = 3$ primary higher spin multiplet

For general superspin Δ with nontrivial representation α for the $SO(3)$, the $\mathcal{N} = 3$ (higher spin) multiplet can be described as

$$\begin{aligned}
\Phi_{\Delta}^{\alpha}(Z) &= \frac{i}{2} \psi_{\Delta}^{\alpha}(z) + \theta^i \frac{i}{2} \phi_{\Delta+\frac{1}{2}}^{i,\alpha}(z) + \theta^{3-i} \frac{1}{2} \psi_{\Delta+1}^{i,\alpha}(z) + \theta^{3-0} \phi_{\Delta+\frac{3}{2}}^{\alpha}(z) \\
&= \frac{i}{2} \psi_{\Delta}^{\alpha}(z) + \theta^1 \frac{i}{2} \phi_{\Delta+\frac{1}{2}}^{1,\alpha}(z) + \theta^2 \frac{i}{2} \phi_{\Delta+\frac{1}{2}}^{2,\alpha}(z) + \theta^3 \frac{i}{2} \phi_{\Delta+\frac{1}{2}}^{3,\alpha}(z) \\
&\quad + \theta^1 \theta^2 \frac{1}{2} \psi_{\Delta+1}^{3,\alpha}(z) + \theta^2 \theta^3 \frac{1}{2} \psi_{\Delta+1}^{1,\alpha}(z) + \theta^3 \theta^1 \frac{1}{2} \psi_{\Delta+1}^{2,\alpha}(z) + \theta^1 \theta^2 \theta^3 \phi_{\Delta+\frac{3}{2}}^{\alpha}(z) \\
&\equiv \left(\frac{i}{2} \psi_{\Delta}^{\alpha}, \quad \frac{i}{2} \phi_{\Delta+\frac{1}{2}}^{i,\alpha}, \quad \frac{1}{2} \psi_{\Delta+1}^{i,\alpha}, \quad \phi_{\Delta+\frac{3}{2}}^{\alpha} \right)
\end{aligned} \tag{2.3} \{?\}$$

In components, there exist a single higher spin- Δ current $\psi_{\Delta}^{\alpha}(z)$, three higher spin- $(\Delta + \frac{1}{2})$ currents $\phi_{\Delta+\frac{1}{2}}^{i,\alpha}(z)$ transforming as a vector representation under $SO(3)$, three higher spin- $(\Delta+1)$ currents $\psi_{\Delta+1}^{i,\alpha}(z)$ transforming as a vector representation under $SO(3)$, and the higher spin- $(\Delta + \frac{3}{2})$ currents $\phi_{\Delta+\frac{3}{2}}^{\alpha}(z)$ for given α representation.³ Depending on the superspin Δ , the above $\mathcal{N} = 3$ (higher spin) multiplet is a bosonic higher spin current for integer spin Δ or a fermionic higher spin current for half-integer spin Δ .

Because the superspin of $\mathbf{J}(Z_1)$ is $\frac{1}{2}$, the right-hand side of the OPE $\mathbf{J}(Z_1) \Phi_{\Delta}^{\alpha}(Z_2)$ has a superspin $(\Delta + \frac{1}{2})$. The pole structure of the linear term in $\Phi_{\Delta}^{\alpha}(Z_2)$ on the right-hand side should have spin $\frac{1}{2}$ without any $SO(3)$ indices. This implies that the structure should be $\frac{\theta_{12}^{3-0}}{z_{12}^2}$, where the spin of $\frac{1}{z_{12}}$ is equal to 1 and the spin of θ_{12} is equal to $-\frac{1}{2}$. The ordinary derivative term can occur at the singular term $\frac{\theta_{12}^{3-0}}{z_{12}^2}$. Furthermore, the spinor derivative terms (descendant terms) with a quadratic product of θ_{12} arise. One should also consider the $\mathcal{N} = 3$ higher spin multiplet $\Phi_{\Delta}^{\beta}(Z_2)$ with different index β with the contraction of $SO(3)$ generator \mathbf{T}^i .⁴ Due to the $SO(3)$ index i of \mathbf{T}^i , we should consider the linear θ_{12}^i term which contracts with the above \mathbf{T}^i . Finally, we obtain the following $\mathcal{N} = 3$ primary condition for the 8 higher spin currents in the $\mathcal{N} = 3$ superspace [23, 25]:

$$\begin{aligned}
\mathbf{J}(Z_1) \Phi_{\Delta}^{\alpha}(Z_2) &= \frac{\theta_{12}^{3-0}}{z_{12}^2} \Delta \Phi_{\Delta}^{\alpha}(Z_2) + \frac{\theta_{12}^{3-i}}{z_{12}} \frac{1}{2} D^i \Phi_{\Delta}^{\alpha}(Z_2) + \frac{\theta_{12}^{3-0}}{z_{12}} \partial \Phi_{\Delta}^{\alpha}(Z_2) \\
&\quad - \frac{\theta_{12}^i}{z_{12}} \frac{i}{2} (\mathbf{T}^i)^{\alpha\beta} \Phi_{\Delta}^{\beta}(Z_2) + \dots
\end{aligned} \tag{2.4} \{?\}$$

where $\theta_{12}^{3-0} = \theta_{12}^1 \theta_{12}^2 \theta_{12}^3$. By assuming that there exist four types of OPEs between the four component currents and the spin- $\frac{1}{2}$ current, $\Psi(z) \psi_{\Delta}^{\alpha}(w)$, $J^i(z) \psi_{\Delta}^{\alpha}(w)$, $G^i(z) \psi_{\Delta}^{\alpha}(w)$, and

³It will turn out that we obtain the $SO(3)$ vector representation for the index α .

⁴We have them explicitly as follows: $\mathbf{T}^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$, $\mathbf{T}^2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}$, $\mathbf{T}^3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

with the relation $[\mathbf{T}^i, \mathbf{T}^j] = i\epsilon^{ijk} \mathbf{T}^k$.

$T(z) \psi_{\Delta}^{\alpha}(w)$, we can write down the $\mathcal{N} = 3$ OPE in (2.4) along the line of the footnote 2. By simply taking $\theta_1^i = \theta_2^i = 0$ in the equation (2.4), we observe that the right hand side vanishes which can be seen from the OPE $\Psi(z) \psi_{\Delta}^{\alpha}(w)$ which is regular in Appendix B. The second term of the right hand side of (2.4) by acting the differential operator D_1^{3-i} and setting $\theta_1^i = \theta_2^i = 0$ can be obtained from the OPE $G^i(z) \psi_{\Delta}^{\alpha}(w)$ in Appendix B. Similarly, by acting $D_1^1 D_1^2 D_1^3$ on the equation (2.4) and putting $\theta_1^i = \theta_2^i = 0$, the singular terms can be obtained from the OPE $T(z) \psi_{\Delta}^{\alpha}(w)$ in Appendix B. Finally, by acting D_1^i on the equation (2.4) and putting $\theta_1^i = \theta_2^i = 0$, the singular term of the last term in (2.4) can be seen from the OPE $J^i(z) \psi_{\Delta}^{\alpha}(w)$ in Appendix B.

We use the following notations for the $SO(3)$ -singlet and $SO(3)$ -triplet $\mathcal{N} = 3$ higher spin multiplet respectively as follows:

$$\Phi_{\Delta}^{\alpha=0}(Z) \rightarrow \Phi^{(\Delta)}(Z), \quad \Phi_{\Delta}^{\alpha}(Z) \rightarrow \Phi^{(\Delta),\alpha}(Z). \quad (2.5) \{?\}$$

Furthermore, we have the component currents, from (2.3) and (2.5),

$$\begin{aligned} \Phi^{(\frac{3}{2})}(Z) &= \frac{i}{2} \psi^{(\frac{3}{2})}(z) + \theta^i \frac{i}{2} \phi^{(2),i}(z) + \theta^{3-i} \frac{1}{2} \psi^{(\frac{5}{2}),i}(z) + \theta^{3-0} \phi^{(3)}(z), \\ \Phi^{(2)}(Z) &= \frac{i}{2} \psi^{(2)}(z) + \theta^i \frac{i}{2} \phi^{(\frac{5}{2}),i}(z) + \theta^{3-i} \frac{1}{2} \psi^{(3),i}(z) + \theta^{3-0} \phi^{(\frac{7}{2})}(z), \\ \Phi^{(\frac{5}{2})}(Z) &= \frac{i}{2} \psi^{(\frac{5}{2})}(z) + \theta^i \frac{i}{2} \phi^{(3),i}(z) + \theta^{3-i} \frac{1}{2} \psi^{(\frac{7}{2}),i}(z) + \theta^{3-0} \phi^{(4)}(z), \\ \Phi^{(3),\alpha}(Z) &= \frac{i}{2} \psi^{(3),\alpha}(z) + \theta^i \frac{i}{2} \phi^{(\frac{7}{2}),i,\alpha}(z) + \theta^{3-i} \frac{1}{2} \psi^{(4),i,\alpha}(z) + \theta^{3-0} \phi^{(\frac{9}{2}),\alpha}(z). \end{aligned} \quad (2.6) \{?\}$$

We will see that the OPE between the first higher spin multiplet and itself leads to the right hand side containing the remaining higher spin multiplets in (2.6) only. We expect that the other higher spin multiplets beyond the above ones will appear in the OPEs between the next lowest higher spin multiplets.

2.3 The realization of $\mathcal{N} = 3$ superconformal algebra

As described in the introduction, we follow the notations used in [18]. The $\alpha = 1, 2, \dots, N^2 - 1$ stands for the adjoint representation of $SU(N)$. The $a = 1, 2, \dots, N$ stands for the fundamental representation of $SU(N)$. Similarly, $\bar{b} = 1, 2, \dots, N$ stands for the anti-fundamental representation of $SU(N)$. The $\rho = 1, 2, \dots, M^2 - 1$ denotes the adjoint representation of $SU(M)$, the $i = 1, 2, \dots, M$ denotes the fundamental representation of $SU(M)$ and the $\bar{j} = 1, 2, \dots, N$ denotes the anti-fundamental representation of $SU(M)$. The OPEs between the spin- $\frac{1}{2}$ currents and the spin-1 currents are presented in Appendix C. The adjoint fermions

of $SU(N+M)$ are denoted by $\Psi^\alpha(z)$, $\Psi^{a\bar{i}}(z)$, $\Psi^{\bar{a}i}(z)$, $\Psi^\rho(z)$ and $\Psi^{u(1)}(z)$ while the vector representation fermions of $SO(2NM)$ are denoted by $\psi^{a\bar{i}}(z)$ and $\psi^{\bar{a}i}(z)$.

The realization of the $\mathcal{N} = 3$ (linear) superconformal algebra has been obtained in [18] and is summarized by

$$\begin{aligned}
\Psi(z) &= \sqrt{\frac{NM}{2}} \Psi^{u(1)}(z), \\
J^1(z) &= \frac{1}{2} \left(\delta_{a\bar{b}} \delta_{\bar{i}j} \Psi^{a\bar{i}} \psi^{\bar{b}j} + \delta_{a\bar{b}} \delta_{\bar{i}j} \psi^{a\bar{i}} \Psi^{\bar{b}j} \right) (z), \\
J^2(z) &= -\frac{i}{2} \left(\delta_{a\bar{b}} \delta_{\bar{i}j} \Psi^{a\bar{i}} \psi^{\bar{b}j} - \delta_{a\bar{b}} \delta_{\bar{i}j} \psi^{a\bar{i}} \Psi^{\bar{b}j} \right) (z), \\
J^3(z) &= \frac{1}{2} \delta_{a\bar{b}} \delta_{\bar{i}j} \left(\Psi^{a\bar{i}} \Psi^{\bar{b}j} - \psi^{a\bar{i}} \psi^{\bar{b}j} \right) (z), \\
G^1(z) &= \frac{1}{\sqrt{2(N+M)}} \left(\delta_{a\bar{b}} \delta_{\bar{i}j} J^{a\bar{i}} \psi^{\bar{b}j} + \delta_{a\bar{b}} \delta_{\bar{i}j} J^{\bar{a}i} \psi^{b\bar{j}} \right) (z), \\
G^2(z) &= -\frac{i}{\sqrt{2(N+M)}} \left(\delta_{a\bar{b}} \delta_{\bar{i}j} J^{a\bar{i}} \psi^{\bar{b}j} - \delta_{a\bar{b}} \delta_{\bar{i}j} J^{\bar{a}i} \psi^{b\bar{j}} \right) (z), \\
G^3(z) &= \frac{1}{\sqrt{2(N+M)}} \left(\Psi^\alpha (J_2^\alpha - j^\alpha) + \Psi^\rho (J_2^\rho - j^\rho) \right) + \frac{1}{\sqrt{2NM}} \Psi^{u(1)} (\hat{J}^{u(1)} - j^{u(1)})(z), \\
T(z) &= \frac{1}{4(N+M)} \left(J^\alpha J^\alpha + J^\rho J^\rho + J^{a\bar{i}} J^{\bar{a}i} + J^{\bar{a}i} J^{a\bar{i}} + J^{u(1)} J^{u(1)} \right) \\
&\quad - \frac{1}{2} \delta_{a\bar{b}} \delta_{\bar{i}j} \left(\psi^{a\bar{i}} \partial \psi^{\bar{b}j} - \partial \psi^{a\bar{i}} \psi^{\bar{b}j} \right) (z) - \frac{1}{4(N+M)} (J^\alpha + j^\alpha)(J^\alpha + j^\alpha)(z) \\
&\quad - \frac{1}{4(N+M)} (J^\rho + j^\rho)(J^\rho + j^\rho)(z) - \frac{1}{4NM} (\hat{J}^{u(1)} + j^{u(1)})(\hat{J}^{u(1)} + j^{u(1)})(z).
\end{aligned} \tag{2.7} \{?\}$$

Among these currents in (2.7), the $\mathcal{N} = 3$ supersymmetry spin- $\frac{3}{2}$ currents $G^i(z)$ are used frequently in order to determine the 8 higher spin currents in next section. The various spin-1 currents are defined in Appendix C.

3 The lowest eight higher spin currents

We would like to construct the lowest eight higher spin currents in terms of coset fermions in the spirit of [52, 53, 54, 55, 56, 57]. We have checked that there is no nontrivial higher spin-1 current.

3.1 The higher spin- $\frac{3}{2}$ current

Let us consider the following ansatz for the lowest higher spin- $\frac{3}{2}$ current, based on the several (N, M) cases,

$$\bar{\psi}^{(\frac{3}{2})}(z) = \left[a_1 J_1^\alpha \Psi^\alpha + a_2 J_2^\alpha \Psi^\alpha + a_3 j^\alpha \Psi^\alpha + b_1 J_1^\rho \Psi^\rho + b_2 J_2^\rho \Psi^\rho + b_3 j^\rho \Psi^\rho \right](z). \quad (3.1) \{?\}$$

The spin-1 currents in terms of fermions are presented in Appendix C. The relative coefficients should be determined. We will determine the normalized lowest higher spin- $\frac{3}{2}$ current later. This ansatz should satisfy the $\mathcal{N} = 3$ primary conditions and the regular (with denominator currents in the coset model) conditions. One of the $\mathcal{N} = 3$ primary conditions (that is, the fifth equation of Appendix (B.1)) is given by

$$J^i(z) \bar{\psi}^{(\frac{3}{2})}(w) = + \dots. \quad (3.2) \{?\}$$

This condition (3.2) requires the relations between the coefficients $a_3 = a_2$ and $b_3 = b_2$. Then the above expression (3.1) can be written as

$$\begin{aligned} \bar{\psi}^{(\frac{3}{2})}(z) &= b_2 \left(\frac{a_2}{b_2} \left[\frac{a_1}{a_2} J_1^\alpha \Psi^\alpha + J_2^\alpha \Psi^\alpha + j^\alpha \Psi^\alpha \right] + \left[\frac{b_1}{b_2} J_1^\rho \Psi^\rho + J_2^\rho \Psi^\rho + j^\rho \Psi^\rho \right] \right)(z) \\ &\sim d \left(a J_1^\alpha \Psi^\alpha + J_2^\alpha \Psi^\alpha + j^\alpha \Psi^\alpha \right)(z) + \left(b J_1^\rho \Psi^\rho + J_2^\rho \Psi^\rho + j^\rho \Psi^\rho \right)(z). \end{aligned} \quad (3.3) \{?\}$$

When we ignore the overall factor b_2 , then we have three unknown coefficients $a(N, M)$, $b(N, M)$, and $d(N, M)$ we should fix.

On the other hands, the regular conditions are given by

$$\begin{aligned} (J_1^\alpha + J_2^\alpha + j^\alpha)(z) \bar{\psi}^{(\frac{3}{2})}(w) &= + \dots, \\ (J_1^\rho + J_2^\rho + j^\rho)(z) \bar{\psi}^{(\frac{3}{2})}(w) &= + \dots. \end{aligned} \quad (3.4) \{?\}$$

Note that the OPE between the expression (3.1) and the $U(1)$ current of the denominator in the coset (1.1) is regular from Appendix C. Then we can determine the coefficients $a(N, M)$ and $b(N, M)$, using the conditions (3.4), as follows:

$$a(N, M) = -\frac{2M}{3N}, \quad b(N, M) = -\frac{2N}{3M}. \quad (3.5) \{?\}$$

There is a $N \leftrightarrow M$ symmetry between the two coefficients in (3.5).

The OPE between the lowest higher spin- $\frac{3}{2}$ current and itself, via the OPEs in Appendix C, can be obtained as follows:

$$\bar{\psi}^{(\frac{3}{2})}(z) \bar{\psi}^{(\frac{3}{2})}(w) = \frac{1}{(z-w)^3} \left[d^2 \frac{2M}{3N} (2M+3N)(N^2-1) + \frac{2N}{3M} (2N+3M)(M^2-1) \right]$$

$$\begin{aligned}
& + \frac{1}{(z-w)} \left[d^2 \frac{2M}{3N} (2M+3N) \partial \Psi^\alpha \Psi^\alpha + d^2 \frac{4M^2}{3N^2} J_1^\alpha J_1^\alpha \right. \\
& + d^2 \left(-\frac{2(2M+N)}{N} J_1^\alpha + J_2^\alpha + j^\alpha \right) (J_2^\alpha + j^\alpha) + \frac{2N}{3M} (2N+3M) \partial \Psi^\rho \Psi^\rho \\
& \left. + \frac{4N^2}{3M^2} J_1^\rho J_1^\rho + \left(-\frac{2(2N+M)}{M} J_1^\rho + J_2^\rho + j^\rho \right) (J_2^\rho + j^\rho) \right] (w) + \dots \quad (3.6) \{?\}
\end{aligned}$$

The normalized higher spin- $\frac{3}{2}$ current can be determined by requiring that the central term should behave as $\frac{2c}{3}$ where the central charge c is given by (1.2)

$$\psi^{(\frac{3}{2})}(z) = \sqrt{\frac{NM}{d^2 \frac{2M}{3N} (2M+3N)(N^2-1) + \frac{2N}{3M} (2N+3M)(M^2-1)}} \bar{\psi}^{(\frac{3}{2})}(z). \quad (3.7) \{?\}$$

By substituting (3.7) into (3.6), we obtain

$$\psi^{(\frac{3}{2})}(z) \psi^{(\frac{3}{2})}(w) = \frac{1}{(z-w)^3} \frac{2}{3} c + \text{other singular terms} + \dots \quad (3.8) \{?\}$$

Furthermore, the lowest higher spin-2 current of the next higher spin multiplet can be obtained from the following expression which will be described in next section

$$\begin{aligned}
\psi^{(2)}(w) = \frac{1}{C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}} & \left[\psi^{(\frac{3}{2})}(z) \psi^{(\frac{3}{2})}(w) \Big|_{\frac{1}{(z-w)}} + \frac{6c}{(c+1)(2c-3)} J^i J^i(w) \right. \\
& \left. + \frac{6(c+3)}{(c+1)(2c-3)} \partial \Psi \Psi(w) - \frac{4c(c+3)}{(c+1)(2c-3)} T(w) \right]. \quad (3.9) \{?\}
\end{aligned}$$

The first term in (3.9) can be read off from (3.6) or (3.8). We can determine the unknown coefficient $d(N, M)$ by using the various $\mathcal{N} = 3$ primary conditions of the next higher spin multiplet including $\psi^{(2)}(z)$. In particular, we have used the fact that $G^+(z) \psi^{(2)}(w) \Big|_{\frac{1}{(z-w)^2}} = 0$ from Appendix B. See also (3.13). The second-order pole of this OPE contains the cubic fermions with the specific index structure. Then by focusing on the coefficient appearing in the particular independent term, we finally obtain the coefficient appearing in (3.3)

$$d^2(N, M) = \frac{N(3M+2N)}{M(3N+2M)}. \quad (3.10) \{?\}$$

By choosing the positive solution in (3.10), the normalized lowest higher spin- $\frac{3}{2}$ current is given by

$$\begin{aligned}
\psi^{(\frac{3}{2})}(z) & = \sqrt{\frac{3N^2M}{2(M+N)(2M+3N)(MN-1)}} \left(-\frac{2M}{3N} J_1^\alpha + J_2^\alpha + j^\alpha \right) \Psi^\alpha(z) \\
& + \sqrt{\frac{3M^2N}{2(M+N)(2N+3M)(MN-1)}} \left(-\frac{2N}{3M} J_1^\rho + J_2^\rho + j^\rho \right) \Psi^\rho(z). \quad (3.11) \{?\}
\end{aligned}$$

We observe that under the exchange of $N \leftrightarrow M$ and $\alpha \leftrightarrow \rho$ this higher spin- $\frac{3}{2}$ current is invariant. This symmetry also appears in the coset (1.1). Some of the terms in (3.11) appear in the spin- $\frac{3}{2}$ current $G^3(z)$ in (2.7). In next subsection, we can obtain the remaining seven higher spin currents with the information of (3.11).

3.2 The higher spin-2 currents

Due to the $\mathcal{N} = 3$ supersymmetry, we can determine other higher spin currents from the lowest one and the spin- $\frac{3}{2}$ currents in (2.7). Using the following relation (coming from Appendix B),

$$G^i(z) \psi^{(\frac{3}{2})}(w) = \frac{1}{(z-w)} \phi^{(2),i}(w) + \dots, \quad (3.12) \{?\}$$

we can obtain three higher spin-2 currents by calculating the left hand side of (3.12) together with (2.7) and (3.11). Note that

$$G^\pm(z) \equiv \frac{1}{\sqrt{2}}(G^1 \pm iG^2)(z), \quad \phi^{(2),\pm}(z) \equiv \frac{1}{\sqrt{2}}(\phi^{(2),1} \pm i\phi^{(2),2})(z). \quad (3.13) \{?\}$$

It is better to use the base (3.13) because $G^\pm(z)$ have simple term rather than $G^1(z)$ or $G^2(z)$ from (2.7). The former will take less time to calculate the OPE manually. By selecting the first-order pole of this OPE, we obtain the higher spin-2 currents $\phi^{(2),\pm}(w)$ directly. The remaining higher spin-2 current $\phi^{(2),3}(w)$ can be obtained from the OPE (3.12). We present the final expression in Appendix (D.2).

3.3 The higher spin- $\frac{5}{2}$ currents

Because we have obtained the higher spin-2 currents in previous subsection, we can continue to calculate the next higher spin-currents. The other defining relation in Appendix B leads to the following expression

$$G^i(z) \phi^{(2),j}(w) \Big|_{\frac{1}{(z-w)}} = \left(\delta^{ij} \partial \psi^{(\frac{3}{2})} + i \epsilon^{ijk} \psi^{(\frac{5}{2}),k} \right)(w). \quad (3.14) \{?\}$$

Let us introduce the following quantities as before

$$\psi^{(\frac{5}{2}),\pm}(z) \equiv \frac{1}{\sqrt{2}}(\psi^{(\frac{5}{2}),1} \pm i\psi^{(\frac{5}{2}),2})(z). \quad (3.15) \{?\}$$

We will see that these preserve the $U(1)$ charge of $\mathcal{N} = 2$ superconformal algebra later. By using (3.14) and (3.15), we can write down the higher spin- $\frac{5}{2}$ currents, together with (2.7),

(3.13) , Appendix (D.2) and (3.11), as follows:

$$\begin{aligned}
\psi^{(\frac{5}{2}),+}(w) &= -G^+(z) \phi^{(2),3}(w) \Big|_{\frac{1}{(z-w)}}, \\
\psi^{(\frac{5}{2}),-}(w) &= G^-(z) \phi^{(2),3}(w) \Big|_{\frac{1}{(z-w)}}, \\
\psi^{(\frac{5}{2}),3}(w) &= G^+(z) \phi^{(2),-}(w) \Big|_{\frac{1}{(z-w)}} - \partial \psi^{(\frac{3}{2})}(w).
\end{aligned} \tag{3.16} \{?\}$$

The last term of the last equation in (3.16) can be obtained from (3.11). It turns out that the final results for the higher spin- $\frac{5}{2}$ currents are given in Appendix (D.3).

3.4 The higher spin-3 current

From the relation which can be obtained from Appendix B,

$$G^i(z) \psi^{(\frac{5}{2}),j}(w) \Big|_{\frac{1}{(z-w)}} = \left(2\delta^{ij} \phi^{(3)} + i\epsilon^{ijk} \partial \phi^{(2),k} \right)(w), \tag{3.17} \{?\}$$

we can read off the higher spin-3 current with (3.13) and (3.15) as follows:

$$\phi^{(3)}(w) = \frac{1}{2} G^+(z) \psi^{(\frac{5}{2}),-}(w) \Big|_{\frac{1}{(z-w)}} - \frac{1}{2} \partial \phi^{(2),3}(w). \tag{3.18} \{?\}$$

The last term for the explicit form can be seen from the previous subsection. We summarize this higher spin-3 current in Appendix (D.4).

Therefore, the lowest eight higher spin currents are obtained in terms of various fermions in (1.1). For the next higher spin multiplets, one can obtain the explicit forms in terms of fermions by using the methods in this section.

4 The OPEs between the lowest eight higher spin currents

In this section, we consider the four types of OPEs between the higher spin currents for fixed $(N, M) = (2, 2)$ where the central charge is given by $c = 6$. Based on the results of this section which are valid for $c = 6$ only (although we put the central charge as c), we can go to the $\mathcal{N} = 2$ superspace approach in next section where all the undetermined coefficient functions will be fixed and can be written in terms of the arbitrary central charge.

4.1 The OPE between the higher spin- $\frac{3}{2}$ current and itself

Let us consider the simplest OPE and calculate the following OPE with the description of previous section.

$$\begin{aligned}\psi^{(\frac{3}{2})}(z)\psi^{(\frac{3}{2})}(w) &= \frac{1}{(z-w)^3}\frac{2c}{3} \\ &+ \frac{1}{(z-w)}\left[\frac{1}{(c+1)(2c-3)}\left(-6cJ^iJ^i-6(c+3)\partial\Psi\Psi+4c(c+3)T\right)\right. \\ &\left.+ C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\psi^{(2)}\right](w)+\dots,\end{aligned}\tag{4.1} \{?\}$$

where the normalization for the lowest higher spin- $\frac{3}{2}$ current is fixed as $\frac{2c}{3}$ as in (4.1). Each three term in the first-order pole of (4.1) is a quasiprimary current in which the third-order pole with the stress energy tensor $T(z)$ does not have any singular term. It turns out that we are left with a new primary higher spin-2 current $\psi^{(2)}(w)$ (which is the lowest component of $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(2)}(Z_2)$ in (2.6)) with a structure constant $C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}$.

4.2 The OPEs between the higher spin- $\frac{3}{2}$ current and the higher spin-2 currents

Let us consider the second type of OPE with the preliminary results in previous section where the explicit forms for the higher spin-2 currents are known. It turns out that we have

$$\begin{aligned}\psi^{(\frac{3}{2})}(z)\phi^{(2),i}(w) &= \frac{1}{(z-w)^2}\frac{1}{(2c-3)}\left[6cG^i-18\Psi J^i\right](w) \\ &+ \frac{1}{(z-w)}\left[\frac{1}{3}\partial(\text{pole two})+\frac{1}{(c+1)(2c-3)}\left(6ic(\epsilon^{ijk}J^jG^k-\frac{2}{3}i\partial G^i)\right.\right. \\ &\left.\left.-18(\partial\Psi J^i-\frac{1}{3}\partial(\Psi J^i))\right)-\frac{1}{2}C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\phi^{(\frac{5}{2}),i}\right](w)+\dots,\end{aligned}\tag{4.2} \{?\}$$

where the correct coefficient $\frac{1}{3}$ for the descendant term in the first-order pole of (4.2) is taken. We can further examine the first-order pole in order to write down in terms of the sum of quasiprimary currents. In this case, there are two kinds of quasiprimary currents. In addition to them, there exists a new primary current $\phi^{(\frac{5}{2}),i}(w)$ which belongs to the previous $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(2)}(Z_2)$ in (2.6). At the moment, it is not obvious how the structure constant appearing in the $\phi^{(\frac{5}{2}),i}(w)$ behaves as the one in (4.2) but this will arise automatically after the analysis of Jacobi identity in next section. In other words, that structure constant can be written in terms of the one introduced in (4.1). Note that in the OPE (4.2), the free index i of $SO(3)$ group appears on the right hand side also.

4.3 The OPE between the higher spin- $\frac{3}{2}$ current and the higher spin- $\frac{5}{2}$ currents

Let us consider the third type of OPE which will be the one of the main important results of this paper. Again, based on the previous section, there are known higher spin- $\frac{5}{2}$ currents in terms of coset fermions for fixed $c = 6$. We obtain the following OPE

$$\begin{aligned}
\psi^{(\frac{3}{2})}(z) \psi^{(\frac{5}{2}),i}(w) &= \frac{1}{(z-w)^3} 6J^i(w) - \frac{1}{(z-w)^2} \frac{18}{(2c-3)} \Psi G^i(w) \\
&+ \frac{1}{(z-w)} \left[\frac{1}{4} \partial(\text{pole two}) \right. \\
&+ \frac{1}{(c+1)(c+6)(2c-3)} \left(18i(5c+6)(\epsilon^{ijk} \Psi J^j G^k - \frac{2}{3} i \Psi \partial G^i) \right. \\
&- 18ic(c+2)(\epsilon^{ijk} G^j G^k - \frac{2}{3} i \partial^2 J^i) + 36(c^2 + 3c + 6)(T J^i - \frac{1}{2} \partial^2 J^i) \\
&- 54(c+2)J^i J^j J^j + 72ic(\epsilon^{ijk} \partial J^j J^k - \frac{1}{3} i \partial^2 J^i) \\
&+ \left. 6(c^2 - 17c - 42)(\partial \Psi G^i - \frac{1}{4} \partial(\Psi G^i)) - 72c \partial \Psi \Psi J^i \right) \\
&+ C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{3(c+3)}{5(c-3)c} \Psi \phi^{(\frac{5}{2}),i} + \frac{9(3c-1)}{5(c-3)c} J^i \psi^{(2)} + \frac{3(c-7)}{10(c-3)} \psi^{(3),i} \right) \\
&+ \left. \frac{2}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \phi^{(3),i} + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \psi^{(3),\alpha=i} \right] (w) + \dots
\end{aligned} \tag{4.3} \{?\}$$

After subtracting the descendant term with the coefficient $\frac{1}{4}$, there exist various quasiprimary currents and primary currents as in (4.3). We can easily check the above seven quasiprimary currents do not have any singular term in the third-order pole in the OPE with $T(z)$. Furthermore, there are three primary currents with structure constant $C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}$ whose presence is not clear at the moment. Among them, the higher spin-3 current $\psi^{(3),i}$ belongs to previous $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(2)}(Z_2)$ in (2.6). Furthermore, there exists a primary current with structure constant $C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}$ which will be defined in next OPE soon. This primary current belongs to the $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(\frac{5}{2})}(Z_2)$ in (2.6). Now the crucial point is the presence of the last term with the structure constant $C_{(\frac{3}{2})(\frac{5}{2})}^{(3)}$ we introduce (or define). We can check that the first-order pole subtracted by the descendant term, seven quasiprimary current terms, and four primary current terms (which is given in terms of the coset fermions explicitly), denoted by $\psi^{(3),\alpha=i}$ with above structure constant, transforms as a primary current under the stress energy tensor $T(z)$. Furthermore, the other defining equations, the first, the fifth and the ninth equations, presented in Appendix B are satisfied. Note that the higher spin-3 current $\psi^{(3),\alpha=i}$ belongs to the $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(3),\alpha}(Z_2)$ in (2.6). Because the left hand side has a free index i for $SO(3)$, we identify that the representation α

corresponds to the $SO(3)$ vector index.

4.4 The OPE between the higher spin- $\frac{3}{2}$ current and the higher spin-3 current

Let us describe the fourth type of OPE. Again, based on the previous section, we can calculate the following OPE and it turns out that

$$\begin{aligned}
\psi^{(\frac{3}{2})}(z)\phi^{(3)}(w) &= \frac{1}{(z-w)^4}3\Psi(w) - \frac{1}{(z-w)^3}3\partial\Psi(w) \\
&+ \frac{1}{(z-w)^2}\left[\frac{1}{(c+1)(2c-3)}\left(6(c+3)(T\Psi - \frac{3}{4}\partial^2\Psi) + 36\frac{c(c+1)}{(c+6)}J^iG^i\right.\right. \\
&- \left.\frac{9(13c+18)}{(c+6)}\Psi J^iJ^i\right) + \frac{3}{2c}C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\Psi\psi^{(2)} + C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}\psi^{(\frac{5}{2})}\left.\right](w) \\
&+ \frac{1}{(z-w)}\left[\frac{1}{5}\partial(\text{pole two}) + \frac{1}{(c+1)(2c-3)}\left(-24(c+3)(\partial\Psi T - \frac{1}{5}\partial(\Psi T))\right.\right. \\
&- \left.12c(\partial J^iG^i - \frac{2}{5}\partial(J^iG^i)) + 18(\partial\Psi J^iJ^i - \frac{1}{5}\partial(\Psi J^iJ^i))\right) \\
&+ C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\left(-\frac{9}{2(c-3)}(\partial\Psi\psi^{(2)} - \frac{1}{5}\partial(\Psi\psi^{(2)})) - \frac{3}{2(c-3)}J^i\phi^{(\frac{5}{2}),i}\right. \\
&- \left.\frac{(c-12)}{5(c-3)}\phi^{(\frac{7}{2})}\right) - \frac{1}{4}C_{(\frac{3}{2})(\frac{5}{2})}^{(3)}\phi^{(\frac{7}{2}),i,\alpha=i}\left.\right](w) + \dots.
\end{aligned} \tag{4.4} \{?\}$$

At the second-order pole of (4.4), there are three quasiprimary currents. Furthermore, there exists a primary current $\psi^{(\frac{5}{2})}(w)$ which belongs to the $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(\frac{5}{2})}(Z_2)$ in (2.6) in addition to the composite current which is also primary current. We introduce the structure constant $C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}$. At the first-order pole, we have the descendant term with coefficient $\frac{1}{5}$. There are also three quasiprimary currents. The quasiprimary current appears in the first term with the structure constant $C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}$. The next two terms are primary currents. Finally, the primary higher spin- $\frac{7}{2}$ currents, which are the second components of the $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(3),\alpha}(Z_2)$ in (2.6), appear. There is a summation over the index i .

4.5 The remaining other OPEs

For the remaining OPEs between the higher spin currents, we have checked that they can be written in terms of known currents and known higher spin currents for fixed central charge using the package of [29].

5 The next higher spin currents

The next lowest higher spin-2 current $\psi^{(2)}(w)$ was obtained from (4.1) by looking at the first-order pole. In other words, by subtracting the three quasiprimary currents in the first-order pole from the left hand side of (4.1), we obtain the nontrivial expression which should satisfy the properties in Appendix B. Because we have explicit form for the higher spin-2 current in terms of fermions, we can also calculate the OPE between this higher spin-2 current and itself and this will eventually determine the structure constant $C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}$ together with the normalization for the higher spin-2 current we fix. Of course, for the $(N, M) = (2, 2)$, we have the explicit expression for the higher spin-2 current in terms of previous fermions. As done in section 3, the corresponding remaining 7 higher spin currents residing on the $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(2)}(Z)$ in (2.6) can be obtained using the $\mathcal{N} = 3$ supersymmetry spin- $\frac{3}{2}$ currents.

The next lowest higher spin- $\frac{5}{2}$ current $\psi^{(\frac{5}{2})}(w)$ has been observed from (4.4) by looking at the second-order pole. Again, there are three quasiprimary currents and the composite current with the previous structure constant. Because the left hand side of (4.4) can be calculated from the explicit form from the section 3, we can extract the higher spin- $\frac{5}{2}$ current with the structure constant $C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}$. Furthermore, in order to fix the normalization for the higher spin- $\frac{5}{2}$ current, we should calculate the OPE between this higher spin- $\frac{5}{2}$ current and itself. Then as we did before, the corresponding remaining 7 higher spin currents residing on the $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(\frac{5}{2})}(Z)$ in (2.6) can be determined using the $\mathcal{N} = 3$ supersymmetry spin- $\frac{3}{2}$ currents.

The next lowest higher spin-3 currents $\psi^{(3),\alpha=i}(w)$ were found from (4.3) by looking at the first-order pole. Once again, for each index i , the higher spin-3 current with the structure constant $C_{(\frac{3}{2})(\frac{5}{2})}^{(3)}$ can be determined from the first-order pole of (4.3) in terms of fermions and the algebraic expressions on the right hand side of (4.3). Based on the higher spin-3 currents, the corresponding remaining each 7 higher spin currents residing on the $\mathcal{N} = 3$ higher spin multiplet $\Phi^{(3),\alpha=i}(Z)$ in (2.6) can be determined, in principle, using the previous $\mathcal{N} = 3$ supersymmetry spin- $\frac{3}{2}$ currents.

6 The OPEs between the lowest eight higher spin currents in $\mathcal{N} = 2$ superspace

To obtain the complete OPEs between the 8 higher spin currents in the $\mathcal{N} = 2$ superspace, the complete composite fields appearing in the OPEs should be determined. It is known that

some of the OPEs between the 8 higher spin currents in the component approach are found explicitly for $(N, M) = (2, 2)$. Then we can move to the $\mathcal{N} = 2$ superspace by collecting those OPEs in the component approach and rearranging them in an $\mathcal{N} = 2$ supersymmetric way. So far, all the coefficients in the OPEs are given with fixed N and M . Now we set these coefficients as functions of N and M and use Jacobi identities between the $\mathcal{N} = 2$ currents or higher spin currents. Eventually, we obtain the complete structure constants with arbitrary central charge appearing in the complete OPEs in the $\mathcal{N} = 2$ superspace.

Let us introduce the two $\mathcal{N} = 2$ higher spin currents ⁵

$$\begin{aligned}\mathbf{W}^{(\frac{3}{2})}(Z) &= 2\psi^{(\frac{3}{2})}(z) + \theta(\phi^{(2),1} + i\phi^{(2),2})(z) + \bar{\theta}(-\phi^{(2),1} + i\phi^{(2),2})(z) + \theta\bar{\theta}\psi^{(\frac{5}{2}),3}(z) \\ &\equiv \left(2\psi^{(\frac{3}{2})}, \quad \phi^{(2),1} + i\phi^{(2),2}, \quad -\phi^{(2),1} + i\phi^{(2),2}, \quad \psi^{(\frac{5}{2}),3}\right),\end{aligned}\tag{6.1} \{?\}$$

and

$$\begin{aligned}\mathbf{W}^{(2)}(Z) &= 2\phi^{(2),3}(z) + \theta(-\psi^{(\frac{5}{2}),1} - i\psi^{(\frac{5}{2}),2})(z) + \bar{\theta}(-\psi^{(\frac{5}{2}),1} + i\psi^{(\frac{5}{2}),2})(z) + \theta\bar{\theta}2\phi^{(3)}(z) \\ &\equiv \left(2\phi^{(2),3}, \quad -\psi^{(\frac{5}{2}),1} - i\psi^{(\frac{5}{2}),2}, \quad -\psi^{(\frac{5}{2}),1} + i\psi^{(\frac{5}{2}),2}, \quad 2\phi^{(3)}\right).\end{aligned}\tag{6.2} \{?\}$$

The exact coefficients appearing in the component currents in (6.1) and (6.2) can be fixed from Appendix B (or its $\mathcal{N} = 2$ version). For example, the $\mathcal{N} = 2$ stress energy tensor is given by Appendix (E.1) and the $\mathcal{N} = 2$ primary conditions are given by the first two equations of Appendix (E.4) which determine the above coefficients exactly. As usual, each second component current of (6.1) and (6.2) has $U(1)$ charge +1 while each third component of them has $U(1)$ charge -1. This can be checked from the OPEs between the $J^3(z)$ current of $\mathcal{N} = 2$ superconformal algebra and the corresponding currents above. We would like to construct the three $\mathcal{N} = 2$ OPEs between these $\mathcal{N} = 2$ higher spin currents.

6.1 The OPE between the $\mathcal{N} = 2$ higher spin- $\frac{3}{2}$ current and itself

Let us consider the OPE between $\mathbf{W}^{(\frac{3}{2})}(Z_1)$ and $\mathbf{W}^{(\frac{3}{2})}(Z_2)$. That is, the OPE between the first $\mathcal{N} = 2$ higher spin- $\frac{3}{2}$ multiplet and itself. The corresponding component results for $(N, M) = (2, 2)$ are obtained from section 4. Now one can introduce the arbitrary coefficients in the right hand side of the OPE. Inside of the package [30], one introduces the OPE in Appendix (E.3) for the $\mathcal{N} = 3$ superconformal algebra in $\mathcal{N} = 2$ superspace, the OPEs in

⁵Although we use the same notation for the $\mathcal{N} = 2$ superspace coordinates as $Z(\bar{Z})$ for the $\mathcal{N} = 3$ superspace coordinates, it is understood that in $\mathcal{N} = 2$ superspace, we have $\theta^3 = \bar{\theta}^3 = 0$. One can write down the $\mathcal{N} = 3$ stress energy tensor (2.1) in terms of two $\mathcal{N} = 2$ ones in Appendix (E.1) and Appendix (E.2) as $\mathbf{J}(Z) = \frac{i}{4}\mathbf{T}^{(\frac{1}{2})}(Z) + \theta^3\frac{i}{2}\mathbf{T}(Z)$ with $\theta \equiv \theta^1 - i\theta^2$ and $\bar{\theta} \equiv -\theta^1 - i\theta^2$. Similarly, one sees $\Phi^{(\frac{3}{2})}(Z) = \frac{i}{4}\mathbf{W}^{(\frac{3}{2})}(Z) + \theta^3\frac{i}{4}\mathbf{W}^{(2)}(Z)$.

Appendix (E.4) where the OPEs are given by the two $\mathcal{N} = 2$ currents, $\mathbf{T}(Z_1)$ and $\mathbf{T}^{(\frac{1}{2})}(Z_1)$, and the $\mathcal{N} = 2$ higher spin multiplets: $\mathbf{W}^{(\frac{3}{2})}(Z_2)$ and $\mathbf{W}^{(2)}(Z_2)$ which have explicit component currents in (6.1) and (6.2), $\mathbf{W}^{(2')}(Z_2)$ and $\mathbf{W}^{(\frac{5}{2})}(Z_2)$, $\mathbf{W}^{(\frac{5}{2})'}(Z_2)$ and $\mathbf{W}^{(3)}(Z_2)$, $\mathbf{W}^{(3),\alpha}(Z_2)$ and $\mathbf{W}^{(\frac{7}{2}),\alpha}(Z_2)$. We write down the component currents for the remaining $\mathcal{N} = 2$ higher spin multiplets as follows:

$$\begin{aligned}
\mathbf{W}^{(2')}(Z) &= 2\psi^{(2)}(z) + \theta(\phi^{(\frac{5}{2}),1} + i\phi^{(\frac{5}{2}),2})(z) + \bar{\theta}(-\phi^{(\frac{5}{2}),1} + i\phi^{(\frac{5}{2}),2})(z) + \theta\bar{\theta}\psi^{(3),3}(z) \\
&\equiv \begin{pmatrix} 2\psi^{(2)}, & \phi^{(\frac{5}{2}),1} + i\phi^{(\frac{5}{2}),2}, & -\phi^{(\frac{5}{2}),1} + i\phi^{(\frac{5}{2}),2}, & \psi^{(3),3} \end{pmatrix}, \\
\mathbf{W}^{(\frac{5}{2})}(Z) &= 2\phi^{(\frac{5}{2}),3}(z) + \theta(-\psi^{(3),1} - i\psi^{(3),2})(z) + \bar{\theta}(-\psi^{(3),1} + i\psi^{(3),2})(z) + \theta\bar{\theta}2\phi^{(\frac{7}{2})}(z) \\
&\equiv \begin{pmatrix} 2\phi^{(\frac{5}{2}),3}, & -\psi^{(3),1} - i\psi^{(3),2}, & -\psi^{(3),1} + i\psi^{(3),2}, & 2\phi^{(\frac{7}{2})} \end{pmatrix}, \\
\mathbf{W}^{(\frac{5}{2})'}(Z) &= 2\psi^{(\frac{5}{2})}(z) + \theta(\phi^{(3),1} + i\phi^{(3),2})(z) + \bar{\theta}(-\phi^{(3),1} + i\phi^{(3),2})(z) + \theta\bar{\theta}\psi^{(\frac{7}{2}),3}(z) \\
&\equiv \begin{pmatrix} 2\psi^{(\frac{5}{2})}, & \phi^{(3),1} + i\phi^{(3),2}, & -\phi^{(3),1} + i\phi^{(3),2}, & \psi^{(\frac{7}{2}),3} \end{pmatrix}, \\
\mathbf{W}^{(3)}(Z) &= 2\phi^{(3),3}(z) + \theta(-\psi^{(\frac{7}{2}),1} - i\psi^{(\frac{7}{2}),2})(z) + \bar{\theta}(-\psi^{(\frac{7}{2}),1} + i\psi^{(\frac{7}{2}),2})(z) + \theta\bar{\theta}2\phi^{(4)}(z) \\
&\equiv \begin{pmatrix} 2\phi^{(3),3}, & -\psi^{(\frac{7}{2}),1} - i\psi^{(\frac{7}{2}),2}, & -\psi^{(\frac{7}{2}),1} + i\psi^{(\frac{7}{2}),2}, & 2\phi^{(4)} \end{pmatrix}, \\
\mathbf{W}^{(3),\alpha}(Z) &= 2\psi^{(3),\alpha} + \theta(\phi^{(\frac{7}{2}),1,\alpha} + i\phi^{(\frac{7}{2}),2,\alpha}) + \bar{\theta}(-\phi^{(\frac{7}{2}),1,\alpha} + i\phi^{(\frac{7}{2}),2,\alpha}) + \theta\bar{\theta}\psi^{(4),3,\alpha} \\
&\equiv \begin{pmatrix} 2\psi^{(3),\alpha}, & \phi^{(\frac{7}{2}),1,\alpha} + i\phi^{(\frac{7}{2}),2,\alpha}, & -\phi^{(\frac{7}{2}),1,\alpha} + i\phi^{(\frac{7}{2}),2,\alpha}, & \psi^{(4),3,\alpha} \end{pmatrix}, \\
\mathbf{W}^{(\frac{7}{2}),\alpha}(Z) &= 2\phi^{(\frac{7}{2}),3,\alpha} + \theta(-\psi^{(4),1,\alpha} - i\psi^{(4),2,\alpha}) + \bar{\theta}(-\psi^{(4),1,\alpha} + i\psi^{(4),2,\alpha}) + \theta\bar{\theta}2\phi^{(\frac{9}{2}),\alpha} \\
&\equiv \begin{pmatrix} 2\phi^{(\frac{7}{2}),3,\alpha}, & -\psi^{(4),1,\alpha} - i\psi^{(4),2,\alpha}, & -\psi^{(4),1,\alpha} + i\psi^{(4),2,\alpha}, & 2\phi^{(\frac{9}{2}),\alpha} \end{pmatrix}. \tag{6.3} \{?\}
\end{aligned}$$

As described before, each second component current for the first four higher spin multiplets in (6.3) has $U(1)$ charge +1 and each third component current for them has $U(1)$ charge -1. For the other two higher spin multiplets, the $U(1)$ charge is little different from the above assignments.⁶ Then one can write down $\mathbf{W}^{(\frac{3}{2})}(Z_1)\mathbf{W}^{(\frac{3}{2})}(Z_2)$, $\mathbf{W}^{(\frac{3}{2})}(Z_1)\mathbf{W}^{(2)}(Z_2)$, and $\mathbf{W}^{(2)}(Z_1)\mathbf{W}^{(2)}(Z_2)$, with arbitrary coefficients. From the component results, we can reduce the independent terms appearing on the right hands of these OPEs. If we do not know them, we should include all possible terms with correct spins at each singular terms. We can consider the cubic terms in $\mathcal{N} = 2$ currents, $\mathbf{T}(Z_2)$ and $\mathbf{T}^{(\frac{1}{2})}(Z_2)$ and the linear terms in the $\mathcal{N} = 2$

⁶We list for the higher spin currents having $U(1)$ charge +1 as follows: $(\psi^{(3),\alpha=1} + i\psi^{(3),\alpha=2})(z)$, $(\phi^{(\frac{7}{2}),3,\alpha=1} + i\phi^{(\frac{7}{2}),3,\alpha=2})(z)$, $(\phi^{(\frac{7}{2}),1,\alpha=3} + i\phi^{(\frac{7}{2}),2,\alpha=3})(z)$, $(\psi^{(4),1,\alpha=3} + i\psi^{(4),2,\alpha=3})(z)$, $(\psi^{(4),3,\alpha=1} + i\psi^{(4),3,\alpha=2})(z)$, and $(\phi^{(\frac{9}{2}),\alpha=1} + i\phi^{(\frac{9}{2}),\alpha=2})(z)$. The corresponding higher spin currents with $U(1)$ charge -1 can be obtained by changing each second term of above ones with minus sign. Furthermore, we have the higher spin currents $(\phi^{(\frac{7}{2}),1,\alpha=1} \pm i\phi^{(\frac{7}{2}),1,\alpha=2} \pm i\phi^{(\frac{7}{2}),2,\alpha=1} - \phi^{(\frac{7}{2}),2,\alpha=2})(z)$ with $U(1)$ charge ± 2 . Similarly, there are the higher spin currents $(\psi^{(4),1,\alpha=1} \pm i\psi^{(4),1,\alpha=2} \pm i\psi^{(4),2,\alpha=1} - \phi^{(4),2,\alpha=2})(z)$ with $U(1)$ charge ± 2 .

higher spin multiplets with the additions of the covariant derivatives, D , \bar{D} and the partial derivative ∂ (there are also mixed terms between them). The number of these derivatives (D , \bar{D} and ∂) are constrained to satisfy the correct spin for the composite currents.

By using the Jacobi identity ⁷ between the three (higher spin) currents

$$(\mathbf{T}, \mathbf{W}^{(\frac{3}{2})}, \mathbf{W}^{(\frac{3}{2})}), \quad (\mathbf{T}^{(\frac{1}{2})}, \mathbf{W}^{(\frac{3}{2})}, \mathbf{W}^{(\frac{3}{2})}), \quad (\mathbf{T}, \mathbf{W}^{(\frac{3}{2})}, \mathbf{W}^{(2)}), \quad (\mathbf{T}^{(\frac{1}{2})}, \mathbf{W}^{(\frac{3}{2})}, \mathbf{W}^{(2)}), \quad (6.4) \{?\}$$

all the structure constants which depend on the central charge c are determined except three unknown ones. There are also the Jacobi identities between the higher spin currents, $(\mathbf{T}, \mathbf{W}^{(2)}, \mathbf{W}^{(2)})$ and $(\mathbf{T}^{(\frac{1}{2})}, \mathbf{W}^{(2)}, \mathbf{W}^{(2)})$, but these are satisfied automatically after imposing the above Jacobi identities (6.4). The first OPE can be summarized by

$$\begin{aligned} \mathbf{W}^{(\frac{3}{2})}(Z_1) \mathbf{W}^{(\frac{3}{2})}(Z_2) &= \frac{1}{z_{12}^3} \frac{8c}{3} + \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^3} 12\mathbf{T}(Z_2) \\ &+ \frac{\theta_{12}}{z_{12}^2} \frac{1}{(2c-3)} \left[-24cD\mathbf{T} - 18\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} \right] (Z_2) \\ &+ \frac{\bar{\theta}_{12}}{z_{12}^2} \frac{1}{(2c-3)} \left[24c\bar{D}\mathbf{T} - 18\mathbf{T}^{(\frac{1}{2})}\bar{D}\mathbf{T}^{(\frac{1}{2})} \right] (Z_2) \\ &+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} \left[-\frac{9}{(2c-3)} \mathbf{T}^{(\frac{1}{2})}[D, \bar{D}]\mathbf{T}^{(\frac{1}{2})} + 12\partial\mathbf{T} \right] (Z_2) \\ &+ \frac{1}{z_{12}} \left[\frac{1}{(c+1)(2c-3)} \left(-6(c+3)\partial\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} + 24c\bar{D}\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} \right. \right. \\ &\quad \left. \left. - 24c\mathbf{T}\mathbf{T} - 8c(c+3)[D, \bar{D}]\mathbf{T} - 24c\partial\mathbf{T} \right) + 2C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \mathbf{W}^{(2')} \right] (Z_2) \\ &+ \frac{\theta_{12}}{z_{12}} \left[\frac{1}{(c+1)(2c-3)} \left(-6(2c+3)\partial D\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} - 12c\partial\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} \right. \right. \\ &\quad \left. \left. - 8c(2c+3)\partial D\mathbf{T} + 6c[D, \bar{D}]\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} - 24c\mathbf{T}D\mathbf{T} \right) \right. \\ &\quad \left. + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} D\mathbf{W}^{(2')} \right] (Z_2) \\ &+ \frac{\bar{\theta}_{12}}{z_{12}} \left[\frac{1}{(c+1)(2c-3)} \left(-6(2c+3)\partial\bar{D}\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} - 12c\partial\mathbf{T}^{(\frac{1}{2})}\bar{D}\mathbf{T}^{(\frac{1}{2})} \right. \right. \\ &\quad \left. \left. + 16c^2\partial\bar{D}\mathbf{T} - 6c\bar{D}\mathbf{T}^{(\frac{1}{2})}[D, \bar{D}]\mathbf{T}^{(\frac{1}{2})} - 24c\mathbf{T}\bar{D}\mathbf{T} \right) \right. \\ &\quad \left. + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \bar{D}\mathbf{W}^{(2')} \right] (Z_2) \\ &+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(6c^2\partial[D, \bar{D}]\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} \right. \right. \end{aligned}$$

⁷The outcome of `OPEJacobi` is a double list of operators [58]. It is better to analyze the elements at the end of the list first because the higher spin currents with large spin appear in the beginning of this list while the higher spin currents with small spin appear at the end of this list.

$$\begin{aligned}
& + 18(c+6)\partial\overline{D}\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} - 18(c+6)\partial D\mathbf{T}^{(\frac{1}{2})}\overline{D}\mathbf{T}^{(\frac{1}{2})} \\
& - 9(c-2)(c+3)\partial\mathbf{T}^{(\frac{1}{2})}[D,\overline{D}]\mathbf{T}^{(\frac{1}{2})} - 36c\mathbf{T}\partial\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} \\
& - 108(c+2)\partial\mathbf{T}\mathbf{T} \\
& + 6(2c^3+9c^2-9c-18)\partial^2\mathbf{T} - 36c(c+2)\partial[D,\overline{D}]\mathbf{T} \\
& - 36(c^2+3c+6)\mathbf{T}[D,\overline{D}]\mathbf{T} + 108(c+2)\mathbf{T}\overline{D}\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} \\
& - 18(5c+6)\overline{D}\mathbf{T}\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} - 18(5c+6)D\mathbf{T}\mathbf{T}^{(\frac{1}{2})}\overline{D}\mathbf{T}^{(\frac{1}{2})} \\
& - 144(c+2)c\overline{D}\mathbf{T}D\mathbf{T} - 108(c+2)\mathbf{T}\mathbf{T}\mathbf{T} \Big) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{9(3c-1)}{5(c-3)c} \mathbf{T}\mathbf{W}^{(2')} + \frac{3(c+3)}{10(c-3)c} \mathbf{T}^{(\frac{1}{2})}\mathbf{W}^{(\frac{5}{2})} \right. \\
& \left. - \frac{3(c-7)}{10(c-3)}[D,\overline{D}]\mathbf{W}^{(2')} \right) + \frac{2}{5}C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \mathbf{W}^{(3)} + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \mathbf{W}^{(3),3} \Big] (Z_2) + \dots.
\end{aligned} \tag{6.5} \{?\}$$

As described before, the maximum power of $\mathcal{N} = 2$ currents is given by 3 while the $\mathcal{N} = 2$ higher spin multiplets appear linearly (there are also mixed terms). There are three unknown structure constants in the OPE. Moreover, we describe the above OPE in simple form as

$$[\mathbf{W}^{(\frac{3}{2})}] \cdot [\mathbf{W}^{(\frac{3}{2})}] = [\mathbf{I}] + [\mathbf{W}^{(2')}] + [\mathbf{W}^{(\frac{5}{2})}] + [\mathbf{W}^{(3)}] + [\mathbf{W}^{(3),3}], \tag{6.6} \{?\}$$

where $[\mathbf{I}]$ in (6.6) stands for the $\mathcal{N} = 3$ superconformal family of identity operator. We have seen the presence of $\psi^{(3),\alpha=i}(w)$ in the OPE of (4.3). In particular, for $\alpha = i = 3$, this higher spin-3 current is the first component of the $\mathcal{N} = 2$ higher spin multiplet $\mathbf{W}^{(3),\alpha=3}(Z)$.

6.2 The OPE between the $\mathcal{N} = 2$ higher spin- $\frac{3}{2}$ current and the $\mathcal{N} = 2$ higher spin-2 current

The explicit OPE for this case is given by Appendix (E.7) and can be summarized by

$$[\mathbf{W}^{(\frac{3}{2})}] \cdot [\mathbf{W}^{(2)}] = [\mathbf{I}] + [\mathbf{W}^{(2')}] + [\mathbf{W}^{(\frac{5}{2})}] + [\mathbf{W}^{(\frac{5}{2}')}] + [\mathbf{W}^{(3),1}] + [\mathbf{W}^{(3),2}] + [\mathbf{W}^{(\frac{7}{2}),3}]. \tag{6.7} \{?\}$$

The $\mathcal{N} = 3$ higher spin multiplets, $\mathbf{W}^{(3),1}(Z_2)$ and $\mathbf{W}^{(3),2}(Z_2)$, appear in particular combinations in (6.7). Because the spin of the left hand side is given by $\frac{7}{2}$, we can have the higher spin- $\frac{7}{2}$ current on the right hand side of (6.7). According to (6.3), the three higher spin- $\frac{7}{2}$ currents $\phi^{(\frac{7}{2}),i,\alpha}(z)$ for each α are residing on the second and third components of $\mathbf{W}^{(3),\alpha}(Z)$ and the first component of $\mathbf{W}^{(\frac{7}{2}),\alpha}(Z)$. When we look at the last line of the OPE in Appendix (E.7), we realize that the $\theta_i = \bar{\theta}_i = 0$ projection (after multiplying the derivatives $D_2^1 D_2^2$ (or $[D_2, \overline{D}_2]$) on both sides) will give rise to the sum of the previous higher spin- $\frac{7}{2}$ currents, denoted by $\phi^{(\frac{7}{2}),i,\alpha=i}(w)$ which appeared in the last term of the first-order pole of (4.4). In other words, the α index in (6.7) associated with the last three terms is summed over the

different $\mathcal{N} = 2$ higher spin multiplets and the other dummy index i is hidden in them. That is, $SO(3)$ index i is contained in the second and third components of $\mathbf{W}^{(3),\alpha}(Z)$ and the first component of $\mathbf{W}^{(\frac{7}{2}),\alpha}(Z)$ for fixed α .

6.3 The OPE between the $\mathcal{N} = 2$ higher spin-2 current and itself

The explicit OPE for this case is given by Appendix (E.8) and can be summarized by

$$[\mathbf{W}^{(2)}] \cdot [\mathbf{W}^{(2)}] = [\mathbf{I}] + [\mathbf{W}^{(2')}] + [\mathbf{W}^{(\frac{5}{2})}] + [\mathbf{W}^{(3)}] + [\mathbf{W}^{(3),3}] + [\mathbf{W}^{(\frac{7}{2}),1}] + [\mathbf{W}^{(\frac{7}{2}),2}]. \quad (6.8) \quad \{?\}$$

In this case, the $\mathcal{N} = 3$ higher spin multiplets, $\mathbf{W}^{(\frac{7}{2}),1}(Z_2)$ and $\mathbf{W}^{(\frac{7}{2}),2}(Z_2)$, appear in particular combinations in (6.8). Note that the $\theta, \bar{\theta}$ independent term (or the first component) of $\mathbf{W}^{(2)}(Z)$ in (6.2) contains the $SO(3)$ index 3. Furthermore, the other two $SO(3)$ indices 1 and 2 appear in the second or third component of $\mathbf{W}^{(2)}(Z)$ in (6.2). Then we can understand that the α indices 1 and 2 of the last two terms in (6.8) correspond to those in the second or third component of $\mathbf{W}^{(2)}(Z_2)$ in (6.2) when we select the first component for the $\mathbf{W}^{(2)}(Z_1)$ on the left hand side of (6.8). In the component approach, one can think of the OPE $\phi^{(2),3}(z) \psi^{(\frac{5}{2}),1}(w)$ or the OPE $\phi^{(2),3}(z) \psi^{(\frac{5}{2}),2}(w)$. This will produce $\phi^{(\frac{7}{2}),3,\alpha=1}(w)$ or $\phi^{(\frac{7}{2}),3,\alpha=2}(w)$ respectively. Similarly, if we take the fourth component of $\mathbf{W}^{(2)}(Z_2)$ in (6.2) (there is no $SO(3)$ index) with the same the first component for the $\mathbf{W}^{(2)}(Z_1)$, then the α index 3 in the $\mathbf{W}^{(3),3}(Z_2)$ in (6.8) originates from the above index 3 in the first component of the $\mathbf{W}^{(2)}(Z_1)$. In other words, in the component approach, the OPE $\phi^{(2),3}(z) \phi^{(3)}(w)$ will lead to $\psi^{(3),\alpha=3}(w)$ on the right hand side.

7 The OPEs between the lowest eight higher spin currents in $\mathcal{N} = 3$ superspace

We summarize the previous results in the $\mathcal{N} = 3$ superspace along the line of [59].

7.1 The OPEs between the lowest eight higher spin currents in the component approach

In the previous section, the complete $\mathcal{N} = 2$ OPEs with complete structure constants are fixed. Again, using the package in [30], we can proceed to the component approach where the 36 OPEs are determined completely. They are presented in Appendix F with simplified notation. We might ask whether or not there exists a possibility of having new primary currents in these 36 OPEs. Because we do not check for them from the 8 higher spin currents

in $\mathcal{N} = 3$ Kazama-Suzuki coset model for generic (N, M) manually, we should be careful about the occurrence of new primary currents in the OPEs. However, in the present case, such a feature does not arise. We have confirmed that there are no extra primary currents in the basic 8 OPEs between the higher spin- $\frac{3}{2}$ current and 8 higher spin currents using the WZW currents for several (N, M) values. We believe that these basic 8 OPEs are satisfied even if we try to calculate them manually.

Let us present how we can read off the component result from its $\mathcal{N} = 2$ version. Let us consider the simplest example given in (4.1). Because the first component of $\mathbf{W}^{\frac{3}{2}}(Z)$ is given by $2\psi^{(\frac{3}{2})}(z)$. We can obtain the following OPE. At the final stage, we put $\theta_i = \bar{\theta}_i = 0$ in order to extract the corresponding component OPE. It turns out that the following expression holds, from (6.5),

$$\begin{aligned} \left. \frac{1}{2} \mathbf{W}^{(\frac{3}{2})}(Z_1) \frac{1}{2} \mathbf{W}^{(\frac{3}{2})}(Z_2) \right|_{\theta_i = \bar{\theta}_i = 0} &= \frac{1}{(z-w)^3} \frac{2c}{3} \\ &+ \frac{1}{(z-w)} \left[\left(\frac{1}{2} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \mathbf{W}^{(2')} - \frac{2c(c+3)}{(c+1)(2c-3)} [D, \bar{D}] \mathbf{T} \right. \right. \\ &- \frac{6c}{(c+1)(2c-3)} \mathbf{T} \mathbf{T} + \frac{6c}{(c+1)(2c-3)} \bar{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \\ &- \frac{3(c+3)}{2(c+1)(2c-3)} \partial \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} \\ &\left. \left. - \frac{6c}{(c+1)(2c-3)} \partial \mathbf{T} \right) \right]_{\theta_i = \bar{\theta}_i = 0} (w) + \dots \end{aligned} \quad (7.1) \{?\}$$

In the first-order pole of (7.1), before we take the condition $\theta_i = \bar{\theta}_i = 0$, they are written in terms of $\mathcal{N} = 2$ (higher spin) currents. As we take this condition, each factor current in the composite currents reduces to its component current. Using the relations in Appendix (E.1), (6.1) and (6.3) with the relation $[J^1, J^2](w) = i\partial J^3(w)$, we obtain the previous OPE in (4.1).

In this way we can obtain the remaining 35 OPEs from its three $\mathcal{N} = 2$ OPEs. We present the 36 component OPEs in Appendix F for convenience: Appendix (F.1), Appendix (F.2) and Appendix (F.3).

7.2 The $\mathcal{N} = 3$ description

The final single $\mathcal{N} = 3$ OPE between the $\mathcal{N} = 3$ (higher spin current) multiplet of superspin $\frac{3}{2}$, with the help of Appendix (F.4) and (4.1), can be described as

$$\begin{aligned} \Phi^{(\frac{3}{2})}(Z_1) \Phi^{(\frac{3}{2})}(Z_2) &= \frac{\theta_{12}^{3-0}}{z_{12}^4} 3\mathbf{J}(Z_2) - \frac{1}{z_{12}^3} \frac{c}{6} + \frac{\theta_{12}^{3-i}}{z_{12}^3} 3D^i \mathbf{J}(Z_2) + \frac{\theta_{12}^{3-0}}{z_{12}^3} 6\partial \mathbf{J}(Z_2) \\ &+ \frac{\theta_{12}^{3-i}}{z_{12}^2} \left[-\frac{18}{(2c-3)} \mathbf{J} D^{3-i} \mathbf{J} + 3\partial D^i \mathbf{J} \right] (Z_2) \end{aligned}$$

$$\begin{aligned}
& + \frac{\theta_{12}^i}{z_{12}^2} \frac{1}{(2c-3)} \left[3cD^{3-i}\mathbf{J} - 18\mathbf{J}D^i\mathbf{J} \right] (Z_2) \\
& + \frac{\theta_{12}^{3-0}}{z_{12}^2} \left[\frac{1}{(c+1)(2c-3)} \left(-6(c+3)(D^{3-0}\mathbf{J}\mathbf{J} + \frac{3}{4}\partial^2\mathbf{J}) - \frac{72c(c+1)}{(c+6)}D^i\mathbf{J}D^{3-i}\mathbf{J} \right. \right. \\
& + \left. \left. \frac{36(13c+18)}{(c+6)}\mathbf{J}D^i\mathbf{J}D^i\mathbf{J} \right) + \frac{9}{2}\partial^2\mathbf{J} - \frac{3i}{c}C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\mathbf{J}\Phi^{(2)} + C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}\Phi^{(\frac{5}{2})} \right] (Z_2) \\
& + \frac{1}{z_{12}} \left[\frac{1}{(c+1)(2c-3)} \left(c(c+3)D^{3-0}\mathbf{J} - 6cD^i\mathbf{J}D^i\mathbf{J} - 6(c+3)\partial\mathbf{J}\mathbf{J} \right) \right. \\
& + \left. \frac{1}{2}iC_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\Phi^{(2)} \right] (Z_2) \\
& + \frac{\theta_{12}^i}{z_{12}} \left[\frac{2}{3}\partial\left(\frac{\theta_{12}^i}{z_{12}^2} - \text{term}\right) + \frac{1}{(c+1)(2c-3)} \left(-6c(\epsilon^{ijk}D^j\mathbf{J}D^{3-k}\mathbf{J} + \frac{1}{3}\partial D^{3-i}\mathbf{J}) \right. \right. \\
& + \left. \left. 18(\partial\mathbf{J}D^i\mathbf{J} - \frac{1}{3}\partial(\mathbf{J}D^i\mathbf{J})) \right) + \frac{i}{4}C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}D^i\Phi^{(2)} \right] (Z_2) \\
& + \frac{\theta_{12}^{3-i}}{z_{12}} \left[\frac{3}{4}\partial\left(\frac{\theta_{12}^{3-i}}{z_{12}^2} - \text{term}\right) - \frac{3}{4}\partial^2D^i\mathbf{J} \right. \\
& + \frac{1}{(c+1)(c+6)(2c-3)} \left(-36(5c+6)(\epsilon^{ijk}\mathbf{J}D^j\mathbf{J}D^{3-k}\mathbf{J} + \frac{1}{3}\mathbf{J}\partial D^{3-i}\mathbf{J}) \right. \\
& + \left. 18c(c+2)(\epsilon^{ijk}D^{3-j}\mathbf{J}D^{3-k}\mathbf{J} - \frac{1}{3}\partial^2D^i\mathbf{J}) - 18(c^2+3c+6)(D^{3-0}\mathbf{J}D^i\mathbf{J} \right. \\
& + \left. \frac{1}{2}\partial^2D^i\mathbf{J}) \right. \\
& + \left. 108(c+2)D^i\mathbf{J}D^j\mathbf{J}D^j\mathbf{J} + 36c(2\epsilon^{ijk}\partial D^j\mathbf{J}D^k\mathbf{J} + \frac{1}{3}\partial^2D^i\mathbf{J}) \right. \\
& - \left. 6(c^2-17c-42)(\partial\mathbf{J}D^{3-i}\mathbf{J} - \frac{1}{4}\partial(\mathbf{J}D^{3-i}\mathbf{J})) + 144c\partial\mathbf{J}D^i\mathbf{J} \right) \\
& + \left. C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{3i(c+3)}{5c(c-3)}\mathbf{J}D^i\Phi^{(2)} - \frac{9i(3c-1)}{5c(c-3)}D^i\mathbf{J}\Phi^{(2)} - \frac{3i(c-7)}{20(c-3)}D^{3-i}\Phi^{(2)} \right) \right. \\
& + \left. \frac{1}{5}C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}D^i\Phi^{(\frac{5}{2})} + \frac{1}{2}C_{(\frac{3}{2})(\frac{5}{2})}^{(3)}\Phi^{(3),\alpha=i} \right] (Z_2) \\
& + \frac{\theta_{12}^{3-0}}{z_{12}} \left[\frac{4}{5}\partial\left(\frac{\theta_{12}^{3-0}}{z_{12}^2} - \text{term}\right) - \frac{8}{5}\partial^3\mathbf{J} \right. \\
& + \frac{1}{(c+1)(2c-3)} \left(-24(c+3)(\partial\mathbf{J}D^{3-0}\mathbf{J} - \frac{1}{5}\partial(\mathbf{J}D^{3-0}\mathbf{J})) \right. \\
& - \left. 24c(\partial D^i\mathbf{J}D^{3-i}\mathbf{J} - \frac{2}{5}\partial(D^i\mathbf{J}D^{3-i}\mathbf{J})) + 72(\partial\mathbf{J}D^i\mathbf{J}D^i\mathbf{J} - \frac{1}{5}\partial(\mathbf{J}D^i\mathbf{J}D^i\mathbf{J})) \right) \\
& + \left. C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{9i}{(c-3)}(\partial\mathbf{J}\Phi^{(2)} - \frac{1}{5}\partial(\mathbf{J}\Phi^{(2)})) - \frac{3i}{(c-3)}D^i\mathbf{J}D^i\Phi^{(2)} \right) \right]
\end{aligned}$$

$$- \frac{i(c-12)}{10(c-3)} D^{3-0} \Phi^{(2)} \Big) + \frac{1}{4} C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} D^i \Phi^{(3), \alpha=i} \Big] (Z_2) + \dots \quad (7.2) \{?\}$$

We can easily see that there are consistent $SO(3)$ index contractions with $SO(3)$ -invariant tensors ϵ^{ijk} and δ^{ij} on the right-hand side of the OPE (7.2).⁸

Therefore, the OPE for the lowest 8 higher spin currents in the $\mathcal{N} = 3$ superspace can be characterized by

$$\left[\Phi^{(\frac{3}{2})} \right] \cdot \left[\Phi^{(\frac{3}{2})} \right] = [\mathbf{I}] + \left[\Phi^{(2)} \right] + \left[\Phi^{(\frac{5}{2})} \right] + \theta^{3-i} \left[\Phi^{(3), i} \right], \quad (7.3) \{?\}$$

where $[\mathbf{I}]$ denotes the large $\mathcal{N} = 3$ linear superconformal family of the identity operator. In the last term of (7.3), we put the quadratic fermionic coordinates in order to emphasize that the $SO(3)$ index i is summed in (7.3). In $\mathcal{N} = 3$ superspace, it is clear that the additional $\mathcal{N} = 3$ higher spin-3 multiplet should transform as the triplet of $SO(3)$. The above OPE (7.3) is equivalent to the previous result given by (6.5), Appendix (E.7), and Appendix (E.8) in the $\mathcal{N} = 2$ superspace. Note that the $\mathcal{N} = 3$ multiplet $\Phi^{(\frac{3}{2})}(Z)$ stands for the two $\mathcal{N} = 2$ higher spin multiplets $\mathbf{W}^{(\frac{3}{2})}(Z)$, and $\mathbf{W}^{(2)}(Z)$.⁹

8 The extension of $SO(\mathcal{N} = 3)$ nonlinear Knizhnik Bershadsky algebra

So far we have considered the $\mathcal{N} = 3$ linear superconformal algebra and its extension. Among the eight currents of the $\mathcal{N} = 3$ linear superconformal algebra, we can decouple the spin- $\frac{1}{2}$ current $\Psi(z)$ from the other remaining seven currents, along the lines of [60, 61, 62]. In this section, we would like to obtain the lowest eight higher spin currents and their OPEs after factoring out the above spin- $\frac{1}{2}$ current.

⁸ Let us examine how the $\mathcal{N} = 3$ OPE can be reduced to the corresponding OPE in the component approach. Let us multiply both sides of (7.2) by the operator $D_2^1 D_2^2$ with the condition of $\theta_1^i = 0 = \theta_2^i$. Then the left-hand side is given by $\frac{i}{2} \psi^{(\frac{3}{2})}(z) (-1) \frac{1}{2} \psi^{(\frac{5}{2}), 3}(w)$. In contrast, the right-hand side contains $D_2^1 D_2^2 \frac{\theta_{12}^1 \theta_{12}^2}{z_{12}^2}$ with current-dependent terms and this leads to the singular term $-\frac{1}{(z-w)^2}$. Therefore, the nonlinear second-order term of the OPE $\psi^{(\frac{3}{2})}(z) \psi^{(\frac{5}{2}), 3}(w)$ is given by $\frac{18}{(2c-3)} \mathbf{J} D^1 D^2 \mathbf{J}(Z_2)$ at vanishing θ_2^i . Furthermore, the additional contribution from the $\frac{\theta_{12}^i}{z_{12}^2}$ leads to $-\frac{36}{(2c-3)} \mathbf{J} D^1 D^2 \mathbf{J}(Z_2)$ at vanishing θ_2^i . Then we obtain $-\frac{18}{(2c-3)} \Psi G^3(w)$ which is equal to the particular singular term in the corresponding OPE in Appendix F.

⁹ Let us describe how we can obtain the $\mathcal{N} = 2$ superspace description starting from its $\mathcal{N} = 3$ version in (7.2). Let us focus on the simplest OPE given by (6.5). Setting $\theta_1^3 = \theta_2^3 = 0$ in (7.2) gives rise to $\frac{i}{4} \mathbf{W}^{(\frac{3}{2})}(Z_1) \frac{i}{4} \mathbf{W}^{(\frac{3}{2})}(Z_2)$ on the left-hand side. For the terms θ_{12}^{3-i} with $i = 3$, we do have any nonzero contributions. Let us focus on the third-order pole of (7.2). We have the relation $\frac{\theta_{12}^1 \theta_{12}^2}{z_{12}^3} = \frac{i}{2} \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^3}$ and we obtain that the coefficient of $\frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^3}$ is given by $-\frac{3}{4} \mathbf{T}(Z_2)$ where we used the fact that $D^3 \mathbf{J}(Z_2)$ at vanishing θ_2^3 is equal to $\frac{i}{2} \mathbf{T}(Z_2)$. This provides the corresponding term in (6.5), as we expect. It is straightforward to check the other remaining terms explicitly.

8.1 Knizhnik Bershadsky algebra

According to the OPEs in Appendix A, the spin-1 currents of the $\mathcal{N} = 3$ superconformal algebra do not have any singular terms with the spin- $\frac{1}{2}$ current. Then it is natural to take the spin-1 currents as the previous one $J^i(z)$. For the spin- $\frac{3}{2}$ currents and the spin-2 current, we should obtain the new spin- $\frac{3}{2}$ currents and spin-2 current. It turns out that the new seven currents are described as [24, 63]

$$\begin{aligned}\hat{T}(z) &= T(z) + \frac{3}{2c}\Psi\partial\Psi(z), \\ \hat{G}^i(z) &= G^i(z) - \frac{3}{c}J^i\Psi(z), \\ \hat{J}^i(z) &= J^i(z).\end{aligned}\tag{8.1} \{?\}$$

The relative coefficients appearing on the right hand side of (8.1) can be fixed by requiring that the following conditions should satisfy

$$\begin{aligned}\Psi(z)\hat{T}(w) &= +\dots, \\ \Psi(z)\hat{G}^i(w) &= +\dots, \\ \Psi(z)\hat{J}^i(w) &= +\dots.\end{aligned}\tag{8.2} \{?\}$$

Then we can calculate the OPEs between the new seven currents using (8.1) and they can be summarized by [27, 28]

$$\begin{aligned}\hat{J}^i(z)\hat{J}^j(w) &= \frac{1}{(z-w)^2}\frac{c}{3}\delta^{ij} + \frac{1}{(z-w)}i\epsilon^{ijk}\hat{J}^k(w) + \dots, \\ \hat{J}^i(z)\hat{G}^j(w) &= \frac{1}{(z-w)}i\epsilon^{ijk}\hat{G}^k(w) + \dots, \\ \hat{G}^i(z)\hat{G}^j(w) &= \frac{1}{(z-w)^3}\frac{1}{3}(2c-3)\delta^{ij} + \frac{1}{(z-w)^2}\frac{i(2c-3)}{c}\epsilon^{ijk}\hat{J}^k(w) \\ &\quad + \frac{1}{(z-w)}\left[2\delta^{ij}\hat{T} + i\epsilon^{ijk}\partial\hat{J}^k - \frac{3}{c}\hat{J}^i\hat{J}^j\right](w) + \dots, \\ \hat{T}(z)\hat{J}^i(w) &= \frac{1}{(z-w)^2}\hat{J}^i(w) + \frac{1}{(z-w)}\partial\hat{J}^i(w) + \dots, \\ \hat{T}(z)\hat{G}^i(w) &= \frac{1}{(z-w)^2}\frac{3}{2}\hat{G}^i(w) + \frac{1}{(z-w)}\partial\hat{G}^i(w) + \dots, \\ \hat{T}(z)\hat{T}(w) &= \frac{1}{(z-w)^4}\frac{1}{4}(2c-1) + \frac{1}{(z-w)^2}2\hat{T}(w) + \frac{1}{(z-w)}\partial\hat{T}(w) + \dots.\end{aligned}\tag{8.3} \{?\}$$

Note that there exists a nonlinear term in the OPE between the spin- $\frac{3}{2}$ currents. One can easily see that the above OPEs (8.3) become the one in [27] (where there is a typo in the OPE) if one changes $i\hat{J}^i(z) = J_K^i(z)$ and other currents remain unchanged.

8.2 The extension of Knizhnik Bershadsky current

According to the first OPE in Appendix B, the lowest higher spin current of any $\mathcal{N} = 3$ multiplet does not have any singular terms with the spin- $\frac{1}{2}$ current. Then it is natural to take the lowest higher spin- $\frac{3}{2}$ current as the previous one $\psi^{(\frac{3}{2})}(z)$. Similarly, the second higher spin-2 currents residing the $SO(3)$ -singlet $\mathcal{N} = 3$ multiplet do not have any singular terms due to the second OPE in Appendix B. Then we do not have to modify the higher spin-2 currents.

On the other hand, for the $SO(3)$ -triplet $\mathcal{N} = 3$ higher spin multiplet, the second higher spin- $\frac{7}{2}$ currents do have the singular terms with the spin- $\frac{1}{2}$ current from the second OPE in Appendix B. Therefore we should find the new higher spin- $\frac{7}{2}, 4, \frac{9}{2}$ currents. The final results for the new higher spin currents are given by

$$\begin{aligned}
\hat{\psi}^{(\frac{3}{2})}(z) &= \psi^{(\frac{3}{2})}(z), \\
\hat{\phi}^{(2),i}(z) &= \phi^{(2),i}(z), \\
\hat{\psi}^{(\frac{5}{2}),i}(z) &= \psi^{(\frac{5}{2}),i}(z) - \frac{3}{c}\phi^{(2),i}\Psi(z), \\
\hat{\phi}^{(3)}(z) &= \phi^{(3)}(z) - \frac{3}{2c}\partial\psi^{(\frac{3}{2})}\Psi(z) + \frac{9}{2c}\psi^{(\frac{3}{2})}\partial\Psi(z), \\
\hat{\psi}^{(2)}(z) &= \psi^{(2)}(z), \\
\hat{\phi}^{(\frac{5}{2}),i}(z) &= \phi^{(\frac{5}{2}),i}(z), \\
\hat{\psi}^{(3),i}(z) &= \psi^{(3),i}(z) - \frac{3}{c}\Psi\phi^{(\frac{5}{2}),i}(z), \\
\hat{\phi}^{(\frac{7}{2})}(z) &= \phi^{(\frac{7}{2})}(z) + \frac{3}{2c}\Psi\partial\psi^{(2)}(z) - \frac{6}{c}\partial\Psi\psi^{(2)}(z), \\
\hat{\psi}^{(\frac{5}{2})}(z) &= \psi^{(\frac{5}{2})}(z), \\
\hat{\phi}^{(3),i}(z) &= \phi^{(3),i}(z), \\
\hat{\psi}^{(\frac{7}{2}),i}(z) &= \psi^{(\frac{7}{2}),i}(z) - \frac{3}{c}\Psi\phi^{(3),i}(z), \\
\hat{\phi}^{(4)}(z) &= \phi^{(4)}(z) + \frac{3}{2c}\Psi\partial\psi^{(\frac{5}{2})}(z) - \frac{15}{2c}\partial\Psi\psi^{(\frac{5}{2})}(z), \\
\hat{\psi}^{(3),\alpha}(z) &= \psi^{(3),\alpha}(z), \\
\hat{\phi}^{(\frac{7}{2}),i,\alpha=j}(z) &= \phi^{(\frac{7}{2}),i,\alpha=j}(z) - \frac{3}{c}i\epsilon^{ijk}\Psi\psi^{(3),\alpha=k}(z), \\
\hat{\psi}^{(4),i,\alpha=j}(z) &= \psi^{(4),i,\alpha=j}(z) - \frac{3}{c}\Psi\left(\phi^{(\frac{7}{2}),i,\alpha=j} + \phi^{(\frac{7}{2}),j,\alpha=i} - \delta^{ij}\phi^{(\frac{7}{2}),k,\alpha=k}\right)(z), \\
\hat{\phi}^{(\frac{9}{2}),\alpha=i}(z) &= \phi^{(\frac{9}{2}),\alpha=i}(z) + \frac{3}{2c}i\epsilon^{ijk}\Psi\psi^{(4),j,\alpha=k}(z) + \frac{3}{2c}\Psi\partial\psi^{(3),\alpha=i}(z) - \frac{9}{c}\partial\Psi\psi^{(3),\alpha=i}(z).
\end{aligned} \tag{8.4} \{?\}$$

Of course, these are regular with the spin- $\frac{1}{2}$ current $\Psi(z)$ as in (8.2). The Ψ dependent terms in the third and fourth equations in (8.4) leads to the nonlinear higher spin currents in their

OPEs. Note that for the higher spin currents with α representation the lowest higher spin currents remain unchanged only. See the last four equations of (8.4).

8.3 The OPEs between the eight lowest higher spin currents

Using the explicit expressions in (8.4), we can calculate the OPEs between the higher spin currents. We should reexpress the right hand sides of these OPEs in terms of new $\mathcal{N} = 3$ higher spin currents in addition to the new $\mathcal{N} = 3$ currents.

We summarize the OPEs as follows:

$$\begin{aligned}
\left[\hat{\psi}^{(\frac{3}{2})}\right] \cdot \left[\hat{\psi}^{(\frac{3}{2})}\right] &= [I] + \left[\hat{\psi}^{(2)}\right], \\
\left[\hat{\psi}^{(\frac{3}{2})}\right] \cdot \left[\hat{\phi}^{(2),i}\right] &= [I] + \left[\hat{\phi}^{(\frac{5}{2}),i}\right], \\
\left[\hat{\psi}^{(\frac{3}{2})}\right] \cdot \left[\hat{\psi}^{(\frac{5}{2}),i}\right] &= [I] + \left[\hat{\psi}^{(2)}\right] + \left[\hat{\psi}^{(3),i}\right] + \left[\hat{\phi}^{(3),i}\right] + \left[\hat{\psi}^{(3),\alpha=i}\right], \\
\left[\hat{\psi}^{(\frac{3}{2})}\right] \cdot \left[\hat{\phi}^{(3)}\right] &= [I] + \left[\hat{\psi}^{(\frac{5}{2})}\right] + \left[\hat{\phi}^{(\frac{5}{2}),i}\right] + \left[\hat{\phi}^{(\frac{7}{2})}\right] + \left[\hat{\phi}^{\frac{7}{2},i,\alpha=i}\right], \\
\left[\hat{\phi}^{(2),i}\right] \cdot \left[\hat{\phi}^{(2),j}\right] &= [I] + \delta^{ij} \left[\hat{\psi}^{(2)}\right] + \epsilon^{ijk} \left[\hat{\psi}^{(3),k}\right] + \epsilon^{ijk} \left[\hat{\phi}^{(3),k}\right] + \epsilon^{ijk} \left[\hat{\psi}^{(3),\alpha=k}\right], \\
\left[\hat{\phi}^{(2),i}\right] \cdot \left[\hat{\psi}^{(\frac{5}{2}),j}\right] &= [I] + \delta^{ij} \left[\hat{\psi}^{(\frac{5}{2})}\right] + (\epsilon^{ijk} + \delta^{ij}) \left[\hat{\phi}^{(\frac{5}{2}),k}\right] + \left[\hat{\phi}^{(\frac{5}{2}),i}\right] + \left[\hat{\phi}^{(\frac{5}{2}),j}\right] \\
&\quad + \delta^{ij} \left[\hat{\phi}^{(\frac{7}{2})}\right] + \epsilon^{ijk} \left[\hat{\psi}^{(2)}\right] + \delta^{ij} \left[\hat{\phi}^{(\frac{7}{2}),k,\alpha=k}\right] + \left[\hat{\phi}^{(\frac{7}{2}),i,\alpha=j}\right] + \epsilon^{ijk} \left[\hat{\psi}^{(\frac{7}{2}),k}\right], \\
\left[\hat{\phi}^{(2),i}\right] \cdot \left[\hat{\phi}^{(3)}\right] &= [I] + \left[\hat{\psi}^{(3),i}\right] + \left[\hat{\psi}^{(2)}\right] + \left[\hat{\phi}^{(3),i}\right] + \left[\hat{\psi}^{(3),\alpha=i}\right] + \epsilon^{ijk} \left[\hat{\phi}^{(\frac{5}{2}),k}\right], \\
&\quad + \epsilon^{ijk} \left[\hat{\psi}^{(3),k}\right] + \epsilon^{ijk} \left[\hat{\psi}^{(4),j,\alpha=k}\right], \\
\left[\hat{\phi}^{(\frac{5}{2}),i}\right] \cdot \left[\hat{\phi}^{(\frac{5}{2}),j}\right] &= [I] + (\epsilon^{ijk} + \delta^{ij}) \left[\hat{\psi}^{(2)}\right] + (\epsilon^{ijk} + \delta^{ij}) \left[\hat{\psi}^{(3),k}\right] + \epsilon^{ijk} \left[\hat{\phi}^{(3),k}\right] \\
&\quad + \epsilon^{ijk} \left[\hat{\psi}^{(3),\alpha=k}\right] + \delta^{ij} \left[\hat{\phi}^{(\frac{5}{2}),k}\right] + \left[\hat{\phi}^{(\frac{5}{2}),i}\right] + \left[\hat{\phi}^{(\frac{5}{2}),j}\right] + \left[\hat{\psi}^{(3),i}\right] \\
&\quad + \left[\hat{\psi}^{(3),j}\right] + \delta^{ij} \left[\hat{\phi}^{(4)}\right] + \left[\hat{\psi}^{(4),i,\alpha=j}\right] + \left[\hat{\psi}^{(4),j,\alpha=i}\right] + \left[\hat{\phi}^{(2),i}\hat{\phi}^{(2),j}\right], \\
\left[\hat{\phi}^{(\frac{5}{2}),i}\right] \cdot \left[\hat{\phi}^{(3)}\right] &= [I] + \left[\hat{\phi}^{(\frac{5}{2}),i}\right] + \epsilon^{ijk} \left[\hat{\phi}^{(\frac{5}{2}),k}\right] + \left[\hat{\psi}^{(2)}\right] + \left[\hat{\psi}^{(\frac{7}{2}),i}\right] + \epsilon^{ijk} \left[\hat{\phi}^{(\frac{7}{2}),j,\alpha=k}\right] \\
&\quad + \epsilon^{ijk} \left[\hat{\psi}^{(3),k}\right] + \left[\hat{\phi}^{(\frac{7}{2})}\right] + \left[\hat{\phi}^{(\frac{9}{2}),\alpha=i}\right] + \left[\hat{\psi}^{(\frac{3}{2})}\hat{\phi}^{(2),i}\right], \\
\left[\hat{\phi}^{(3)}\right] \cdot \left[\hat{\phi}^{(3)}\right] &= [I] + \left[\hat{\psi}^{(2)}\right] + \left[\hat{\phi}^{(\frac{5}{2}),i}\right] + \left[\hat{\psi}^{(3),i}\right] + \left[\hat{\phi}^{(4)}\right] + \left[\hat{\psi}^{(4),i,\alpha=i}\right] + \left[\partial\hat{\psi}^{(\frac{3}{2})}\hat{\psi}^{(\frac{3}{2})}\right].
\end{aligned} \tag{8.5} \{?\}$$

Here we use the simplified notations where the indices appearing in the $\mathcal{N} = 3$ (nonlinear) superconformal currents are ignored. This is the reason why the above OPEs (8.5) do not preserve the covariance in the $SO(3)$ indices. In particular, the nonlinear terms between the higher spin currents appear in the last three OPEs of (8.5). It is easy to see that these come from the Ψ dependent terms in the three places of the third and the fourth equations of (8.4).

9 Conclusions and outlook

We have constructed the eight higher spin currents, (3.11), Appendix (D.2), Appendix (D.3) and Appendix (D.4). We have found their OPEs given in (6.5), Appendix (E.7) and Appendix (E.8) or its component results in Appendix (F.1), Appendix (F.2) and Appendix (F.3) or its $\mathcal{N} = 3$ version in (7.2).

Several comments are in order. The Jacobi identities used in section 6 are not used completely because the OPEs between the higher spin currents are not known. We emphasize that those Jacobi identities are exactly zero. As we further study the OPEs between the lowest higher spin- $\frac{3}{2}$ current and the next higher spin currents, we expect that the Jacobi identities are satisfied up to null fields as in [14]. Non-freely generated algebra described in [64] due to the presence of the null fields has been checked in [36] by applying to the Kazama-Suzuki model. See also the previous works [65, 66] in the other specific examples how to use the vacuum character to check the null fields at a given spin.

One might ask whether there is a possibility to have the additional higher spin currents in the right hand side of (7.2). In Appendix H, we present the $(N, M) = (3, 2)$ case where there is NO extra higher spin current appearing in the right hand side of (7.2). Although the vacuum character in the $\mathcal{N} = 3$ Kazama-Suzuki model will be found explicitly (and therefore the spin contents will be known up to a given spin), it will not be easy to see the extra higher spin currents completely as we add them in the right hand side of (7.2). In order to determine the structure constants appearing in these extra higher spin currents, we should use the other Jacobi identities between them we do not know at the moment. Note that the structure constants in (7.2) are not determined.

If the extra higher spin currents are present in the right hand side of (7.2) in the large (N, M) values, we expect that they will appear linearly [1] without spoiling the precise expression in the OPE (7.2). Of course, they can appear in the combinations with the $\mathcal{N} = 3$ stress energy tensor as usual. The structure constants in them should contain the factors $(c - 6)$ corresponding to $(N, M) = (2, 2)$ case, $(c - 9)$ corresponding to $(N, M) = (3, 2)$ case and so on. We may try to calculate the OPEs from Appendix D manually but this is beyond the scope of this paper because it is rather involved to obtain the higher spin currents $\Phi^{(2)}(Z)$, $\Phi^{(\frac{5}{2})}(Z)$ and $\Phi^{(3),\alpha}(Z)$ explicitly with arbitrary (N, M) dependence.

We list some open problems.

- Marginal operator

As in the large c limit, we would like to construct the marginal operator which breaks the higher spin symmetry but keeps the $\mathcal{N} = 3$ superconformal symmetry. It is an open problem

to obtain the eigenvalue equation for the various zero modes acting on the corresponding states and to obtain the mass terms for generic c .

- Orthogonal Kazama-Suzuki model

For the coset in [37]

$$\frac{\hat{SO}(2N+M)_k \oplus \hat{SO}(2NM)_1}{\hat{SO}(2N)_{k+M} \oplus \hat{SO}(M)_{k+2N}}, \quad c = \frac{3kNM}{(M+2N+k-2)}, \quad (9.1) \{?\}$$

the $M = 2$ case in (9.1) corresponds to the other type of Kazama-Suzuki model. What happens for the level $k = 2N$ which is the dual Coxeter number of $SO(2N+2)$? It would be interesting to see whether this coset model makes an enhancement of the original $\mathcal{N} = 2$ supersymmetry and to obtain the higher spin currents (if they exist). The relevant works [67, 68, 69, 70] on this direction will be useful to construct the higher spin currents. The relevant work can be found in [71] where the $\mathcal{N} = 1$ supersymmetry enhancement in the orthogonal coset exists at the critical level.

- Large $\mathcal{N} = 4$ coset theory at the critical level

As described in the introduction, the $M = 2$ case in the coset model (1.1) is similar to the large $\mathcal{N} = 4$ coset theory. It is an open problem to check whether there exists an enhancement of the supersymmetry at the particular critical level or not. It is not known what is the algebra which has an \mathcal{N} supersymmetry with $\mathcal{N} > 4$. Therefore, it is better to study the fermion model having the \mathcal{N} nonlinear superconformal algebra [27, 28]. Then we should add the appropriate spin-1 and spin- $\frac{1}{2}$ currents in order to obtain the linear algebra. It is natural to ask the possibility for the higher spin extension.

- The OPEs between the next higher spin multiplets

So far, we have considered the OPE between the lowest $\mathcal{N} = 3$ higher spin multiplet. It is an open problem to calculate the other OPEs between the next $\mathcal{N} = 3$ higher spin multiplets.

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A The $\mathcal{N} = 3$ superconformal algebra in section 2

We present the $\mathcal{N} = 3$ superconformal algebra in component approach corresponding to (2.2) as follows [21, 22]:

$$\begin{aligned}
\Psi(z) \Psi(w) &= \frac{1}{(z-w)} \frac{c}{3} + \dots, \\
\Psi(z) G^i(w) &= \frac{1}{(z-w)} J^i(w) + \dots, \\
J^i(z) J^j(w) &= \frac{1}{(z-w)^2} \frac{c}{3} \delta^{ij} + \frac{1}{(z-w)} i\epsilon^{ijk} J^k(w) + \dots, \\
J^i(z) G^j(w) &= \frac{1}{(z-w)^2} \delta^{ij} \Psi(w) + \frac{1}{(z-w)} i\epsilon^{ijk} G^k(w) + \dots, \\
G^i(z) G^j(w) &= \frac{1}{(z-w)^3} \frac{2c}{3} \delta^{ij} + \frac{1}{(z-w)^2} 2i\epsilon^{ijk} J^k(w) + \frac{1}{(z-w)} \left[2\delta^{ij} T + i\epsilon^{ijk} \partial J^k \right] (w) + \dots, \\
T(z) \Psi(w) &= \frac{1}{(z-w)^2} \frac{1}{2} \Psi(w) + \frac{1}{(z-w)} \partial \Psi(w) + \dots, \\
T(z) J^i(w) &= \frac{1}{(z-w)^2} J^i(w) + \frac{1}{(z-w)} \partial J^i(w) + \dots, \\
T(z) G^i(w) &= \frac{1}{(z-w)^2} \frac{3}{2} G^i(w) + \frac{1}{(z-w)} \partial G^i(w) + \dots, \\
T(z) T(w) &= \frac{1}{(z-w)^4} \frac{c}{2} + \frac{1}{(z-w)^2} 2T(w) + \frac{1}{(z-w)} \partial T(w) + \dots.
\end{aligned} \tag{A.1} \{?\}$$

Note that there are no singular terms in the OPE between the spin- $\frac{1}{2}$ current $\Psi(z)$ and the spin-1 current $J^i(w)$. One also obtains the OPE $G^i(z) \Psi(w) = \frac{1}{(z-w)} J^i(w) + \dots$. The nonlinear version of above $\mathcal{N} = 3$ superconformal algebra can be obtained by factoring out the spin- $\frac{1}{2}$ current [24, 63] and becomes the result of (8.3).

B The $\mathcal{N} = 3$ primary conditions in the component approach in section 2

From the $\mathcal{N} = 3$ primary higher spin multiplet in (2.4), we write down them in component approach as follows [23]:

$$\begin{aligned}
\Psi(z) \psi_{\Delta}^{\alpha}(w) &= +\dots, \\
\Psi(z) \phi_{\Delta+\frac{1}{2}}^{i,\alpha}(w) &= -\frac{1}{(z-w)} (\mathbf{T}^i)^{\alpha\beta} \psi_{\Delta}^{\beta}(w) + \dots, \\
\Psi(z) \psi_{\Delta+1}^{i,\alpha}(w) &= \frac{1}{(z-w)} \left[\phi_{\Delta+\frac{1}{2}}^{i,\alpha} + i\epsilon^{ijk} (\mathbf{T}^j)^{\alpha\beta} \phi_{\Delta+\frac{1}{2}}^{k,\beta} \right] (w) + \dots, \\
\Psi(z) \phi_{\Delta+\frac{3}{2}}^{\alpha}(w) &= \frac{1}{(z-w)^2} \Delta \psi_{\Delta}^{\alpha}(w) - \frac{1}{(z-w)} \left[\frac{1}{2} \partial \psi_{\Delta}^{\alpha} + \frac{1}{2} (\mathbf{T}^i)^{\alpha\beta} \psi_{\Delta+1}^{i,\beta} \right] (w) + \dots, \\
J^i(z) \psi_{\Delta}^{\alpha}(w) &= -\frac{1}{(z-w)} (\mathbf{T}^i)^{\alpha\beta} \psi_{\Delta}^{\beta}(w) + \dots,
\end{aligned}$$

$$\begin{aligned}
J^i(z) \phi_{\Delta+\frac{1}{2}}^{j,\alpha}(w) &= \frac{1}{(z-w)} \left[i\epsilon^{ijk} \phi_{\Delta+\frac{1}{2}}^{k,\alpha} - (\mathbf{T}^i)^{\alpha\beta} \phi_{\Delta+\frac{1}{2}}^{j,\beta} \right] (w) + \dots, \\
J^i(z) \psi_{\Delta+1}^{j,\alpha}(w) &= \frac{1}{(z-w)^2} \left[2\Delta \delta^{ij} \psi_{\Delta}^{\alpha} - i\epsilon^{ijk} (\mathbf{T}^k)^{\alpha\beta} \psi_{\Delta}^{\beta} \right] (w) \\
&+ \frac{1}{(z-w)} \left[i\epsilon^{ijk} \psi_{\Delta+1}^{k,\alpha} - (\mathbf{T}^i)^{\alpha\beta} \psi_{\Delta+1}^{j,\beta} \right] (w) + \dots, \\
J^i(z) \phi_{\Delta+\frac{3}{2}}^{\alpha}(w) &= \frac{1}{(z-w)^2} \left[\left(\Delta + \frac{1}{2}\right) \phi_{\Delta+\frac{1}{2}}^{i,\alpha} + \frac{i}{2} \epsilon^{ijk} (\mathbf{T}^j)^{\alpha\beta} \phi_{\Delta+\frac{1}{2}}^{k,\beta} \right] (w) \\
&- \frac{1}{(z-w)} (\mathbf{T}^i)^{\alpha\beta} \phi_{\Delta+\frac{3}{2}}^{\beta}(w) + \dots, \\
G^i(z) \psi_{\Delta}^{\alpha}(w) &= \frac{1}{(z-w)} \phi_{\Delta+\frac{1}{2}}^{i,\alpha}(w) + \dots, \\
G^i(z) \phi_{\Delta+\frac{1}{2}}^{j,\alpha}(w) &= \frac{1}{(z-w)^2} \left[2\delta^{ij} \Delta \psi_{\Delta}^{\alpha} - i\epsilon^{ijk} (\mathbf{T}^k)^{\alpha\beta} \psi_{\Delta}^{\beta} \right] (w) \\
&+ \frac{1}{(z-w)} \left[\delta^{ij} \partial \psi_{\Delta}^{\alpha} + i\epsilon^{ijk} \psi_{\Delta+1}^{k,\alpha} \right] (w) + \dots, \\
G^i(z) \psi_{\Delta+1}^{j,\alpha}(w) &= \frac{1}{(z-w)^2} \left[2\left(\Delta + \frac{1}{2}\right) i\epsilon^{ijk} \phi_{\Delta+\frac{1}{2}}^{k,\alpha} + (\mathbf{T}^j)^{\alpha\beta} \phi_{\Delta+\frac{1}{2}}^{i,\beta} - \delta^{ij} (\mathbf{T}^k)^{\alpha\beta} \phi_{\Delta+\frac{1}{2}}^{k,\beta} \right] (w) \\
&+ \frac{1}{(z-w)} \left[2\delta^{ij} \phi_{\Delta+\frac{3}{2}}^{\alpha} + i\epsilon^{ijk} \partial \phi_{\Delta+\frac{1}{2}}^{k,\alpha} \right] (w) + \dots, \\
G^i(z) \phi_{\Delta+\frac{3}{2}}^{\alpha}(w) &= -\frac{1}{(z-w)^3} (\mathbf{T}^i)^{\alpha\beta} \psi_{\Delta}^{\beta}(w) \\
&+ \frac{1}{(z-w)^2} \left[(\Delta + 1) \psi_{\Delta+1}^{i,\alpha} + \frac{i}{2} \epsilon^{ijk} (\mathbf{T}^j)^{\alpha\beta} \psi_{\Delta+1}^{k,\beta} \right] (w) \\
&+ \frac{1}{(z-w)} \frac{1}{2} \partial \psi_{\Delta+1}^{i,\alpha}(w) + \dots, \\
T(z) \psi_{\Delta}^{\alpha}(w) &= \frac{1}{(z-w)^2} \Delta \psi_{\Delta}^{\alpha}(w) + \frac{1}{(z-w)} \partial \psi_{\Delta}^{\alpha}(w) + \dots, \\
T(z) \phi_{\Delta+\frac{1}{2}}^{i,\alpha}(w) &= \frac{1}{(z-w)^2} \left(\Delta + \frac{1}{2}\right) \phi_{\Delta+\frac{1}{2}}^{i,\alpha}(w) + \frac{1}{(z-w)} \partial \phi_{\Delta+\frac{1}{2}}^{i,\alpha}(w) + \dots, \\
T(z) \psi_{\Delta+1}^{i,\alpha}(w) &= -\frac{1}{(z-w)^3} (\mathbf{T}^i)^{\alpha\beta} \psi_{\Delta}^{\beta}(w) + \frac{1}{(z-w)^2} (\Delta + 1) \psi_{\Delta+1}^{i,\alpha}(w) + \frac{1}{(z-w)} \partial \psi_{\Delta+1}^{i,\alpha}(w) + \dots, \\
T(z) \phi_{\Delta+\frac{3}{2}}^{\alpha}(w) &= -\frac{1}{(z-w)^3} \frac{1}{2} (\mathbf{T}^i)^{\alpha\beta} \phi_{\Delta+\frac{1}{2}}^{i,\beta}(w) + \frac{1}{(z-w)^2} \left(\Delta + \frac{3}{2}\right) \phi_{\Delta+\frac{3}{2}}^{\alpha}(w) + \frac{1}{(z-w)} \partial \phi_{\Delta+\frac{3}{2}}^{\alpha}(w) \\
&+ \dots.
\end{aligned} \tag{B.1} \{?\}$$

For the $SO(3)$ singlet $\mathcal{N} = 3$ higher spin multiplet, we ignore the $SO(3)$ generator terms with boldface notation in (B.1). Note that the higher spin currents $\psi_{\Delta+1}^{i,\alpha}(w)$ and $\phi_{\Delta+\frac{3}{2}}^{\alpha}(w)$ are not primary currents under the stress energy tensor $T(z)$ according to the last two equations in (B.1). We obtain the primary higher spin currents $(\psi_{\Delta+1}^{i,\alpha} + \frac{1}{2\Delta} (\mathbf{T}^i)^{\alpha\beta} \partial \psi_{\Delta}^{\beta})(w)$ and $(\phi_{\Delta+\frac{3}{2}}^{\alpha} + \frac{1}{2(2\Delta+1)} (\mathbf{T}^i)^{\alpha\beta} \partial \phi_{\Delta+\frac{1}{2}}^{i,\beta})(w)$.

C The fundamental OPEs between the spin- $\frac{1}{2}$ and the spin-1 currents in section 2

We present the various OPEs between the spin- $\frac{1}{2}$ currents and the spin-1 currents.

C.1 The OPEs between the spin- $\frac{1}{2}$ currents

There are five nontrivial OPEs as follows:

$$\begin{aligned}
\Psi^\alpha(z) \Psi^\beta(w) &= \frac{1}{(z-w)} \delta^{\alpha\beta} + \dots, \\
\Psi^\rho(z) \Psi^\sigma(w) &= \frac{1}{(z-w)} \delta^{\rho\sigma} + \dots, \\
\Psi^{u(1)}(z) \Psi^{u(1)}(w) &= \frac{1}{(z-w)} + \dots, \\
\Psi^{a\bar{i}}(z) \Psi^{\bar{b}j}(w) &= \frac{1}{(z-w)} \delta^{a\bar{b}} \delta^{\bar{i}j} + \dots, \\
\psi^{a\bar{i}}(z) \psi^{\bar{b}j}(w) &= \frac{1}{(z-w)} \delta^{a\bar{b}} \delta^{\bar{i}j} + \dots,
\end{aligned} \tag{C.1} \{?\}$$

where the first four OPEs correspond to (3.18) of [18].

C.2 The OPEs between the spin- $\frac{1}{2}$ currents and the spin-1 currents

By specifying the structure constants explicitly, we have the following OPEs between the spin- $\frac{1}{2}$ currents and the spin-1 currents

$$\begin{aligned}
\Psi^\alpha(z) J_1^\beta(w) &= \frac{1}{(z-w)} i f^{\alpha\beta\gamma} \Psi^\gamma(w) + \dots, \\
\Psi^\alpha(z) J^{a\bar{i}}(w) &= \frac{1}{(z-w)} \Psi^{b\bar{i}} t_{b\bar{a}}^\alpha(w) + \dots, \\
\Psi^\alpha(z) J^{\bar{a}i}(w) &= -\frac{1}{(z-w)} t_{a\bar{b}}^\alpha \Psi^{\bar{b}i}(w) + \dots, \\
\Psi^\rho(z) J_1^\sigma(w) &= \frac{1}{(z-w)} i f^{\rho\sigma\tau} \Psi^\tau(w) + \dots, \\
\Psi^\rho(z) J^{a\bar{i}}(w) &= -\frac{1}{(z-w)} t_{i\bar{j}}^\rho \Psi^{a\bar{j}}(w) + \dots, \\
\Psi^\rho(z) J^{\bar{a}i}(w) &= \frac{1}{(z-w)} \Psi^{\bar{a}j} t_{j\bar{i}}^\rho(w) + \dots, \\
\Psi^{u(1)}(z) J^{a\bar{i}}(w) &= \frac{1}{(z-w)} \sqrt{\frac{N+M}{NM}} \Psi^{a\bar{i}}(w) + \dots, \\
\Psi^{u(1)}(z) J^{\bar{a}i}(w) &= -\frac{1}{(z-w)} \sqrt{\frac{N+M}{NM}} \Psi^{\bar{a}i}(w) + \dots, \\
\Psi^{a\bar{i}}(z) J_2^\alpha(w) &= -\frac{1}{(z-w)} \Psi^{b\bar{i}} t_{b\bar{a}}^\alpha(w) + \dots,
\end{aligned}$$

$$\begin{aligned}
\Psi^{a\bar{i}}(z) J_2^\rho(w) &= \frac{1}{(z-w)} t_{i\bar{j}}^\rho \Psi^{a\bar{j}}(w) + \dots, \\
\Psi^{a\bar{i}}(z) J^{u(1)}(w) &= -\frac{1}{(z-w)} \sqrt{\frac{N+M}{NM}} \Psi^{a\bar{i}}(w) + \dots, \\
\Psi^{b\bar{j}}(z) J^{\bar{a}i}(w) &= \frac{1}{(z-w)} \left(\sqrt{\frac{N+M}{NM}} \delta^{b\bar{a}} \delta^{\bar{j}i} \Psi^{u(1)} + \delta^{\bar{j}i} t_{a\bar{b}}^\alpha \Psi^\alpha - \delta^{b\bar{a}} \Psi^\rho t_{j\bar{i}}^\rho \right) (w) + \dots, \\
\Psi^{\bar{a}i}(z) J_2^\alpha(w) &= \frac{1}{(z-w)} t_{a\bar{b}}^\alpha \Psi^{\bar{b}i}(w) + \dots, \\
\Psi^{\bar{a}i}(z) J_2^\rho(w) &= -\frac{1}{(z-w)} \Psi^{\bar{a}j} t_{j\bar{i}}^\rho(w) + \dots, \\
\Psi^{\bar{a}i}(z) J^{u(1)}(w) &= \frac{1}{(z-w)} \sqrt{\frac{N+M}{NM}} \Psi^{\bar{a}i}(w) + \dots, \\
\Psi^{\bar{b}j}(z) J^{a\bar{i}}(w) &= \frac{1}{(z-w)} \left(-\sqrt{\frac{N+M}{NM}} \delta^{\bar{b}a} \delta^{j\bar{i}} \Psi^{u(1)} - \delta^{j\bar{i}} t_{b\bar{a}}^\alpha \Psi^\alpha + \delta^{\bar{b}a} \Psi^\rho t_{i\bar{j}}^\rho \right) (w) + \dots, \\
\psi^{a\bar{i}}(z) j^\alpha(w) &= -\frac{1}{(z-w)} \psi^{b\bar{i}} t_{b\bar{a}}^\alpha(w) + \dots, \\
\psi^{a\bar{i}}(z) j^\rho(w) &= \frac{1}{(z-w)} t_{i\bar{j}}^\rho \psi^{a\bar{j}}(w) + \dots, \\
\psi^{a\bar{i}}(z) j^{u(1)}(w) &= -\frac{1}{(z-w)} \psi^{a\bar{i}}(w) + \dots, \\
\psi^{\bar{a}i}(z) j^\alpha(w) &= \frac{1}{(z-w)} t_{a\bar{b}}^\alpha \psi^{\bar{b}i}(w) + \dots, \\
\psi^{\bar{a}i}(z) j^\rho(w) &= -\frac{1}{(z-w)} \psi^{\bar{a}j} t_{j\bar{i}}^\rho(w) + \dots, \\
\psi^{\bar{a}i}(z) j^{u(1)}(w) &= \frac{1}{(z-w)} \psi^{\bar{a}i}(w) + \dots,
\end{aligned} \tag{C.2} \{?\}$$

corresponding to (3.20) of [18]. The spin-1 currents are given by

$$\begin{aligned}
J_1^\alpha(z) &\equiv -\frac{i}{2} f^{\alpha\beta\gamma} \Psi^\beta \Psi^\gamma(z), \\
J_2^\alpha(z) &\equiv \delta_{i\bar{j}} \Psi^{b\bar{i}} t_{b\bar{a}}^\alpha \Psi^{a\bar{j}}(z), \\
J_1^\rho(z) &\equiv -\frac{i}{2} f^{\rho\sigma\tau} \Psi^\sigma \Psi^\tau(z), \\
J_2^\rho(z) &= \delta_{a\bar{b}} \Psi^{\bar{a}i} t_{i\bar{j}}^\rho \Psi^{b\bar{j}}(z), \\
J^{u(1)}(z) &\equiv \sqrt{\frac{N+M}{NM}} \delta_{a\bar{b}} \delta_{i\bar{j}} \Psi^{a\bar{i}} \Psi^{b\bar{j}}(z), \\
J^{a\bar{i}}(z) &\equiv \left(\sqrt{\frac{N+M}{NM}} \Psi^{u(1)} \Psi^{a\bar{i}} + \delta^{\bar{a}a} t_{b\bar{a}}^\alpha \Psi^\alpha \Psi^{b\bar{i}} - \Psi^\rho \delta^{\bar{i}i} t_{j\bar{i}}^\rho \Psi^{a\bar{j}} \right) (z), \\
J^{\bar{a}i}(z) &\equiv \left(-\sqrt{\frac{N+M}{NM}} \Psi^{u(1)} \Psi^{\bar{a}i} - \delta^{\bar{a}a} t_{a\bar{b}}^\alpha \Psi^\alpha \Psi^{\bar{b}i} + \Psi^\rho \delta^{\bar{i}i} t_{j\bar{i}}^\rho \Psi^{\bar{a}j} \right) (z), \\
j^\alpha(z) &\equiv \delta_{i\bar{j}} \psi^{b\bar{i}} t_{b\bar{a}}^\alpha \psi^{\bar{a}j}(z),
\end{aligned}$$

$$\begin{aligned}
j^\rho(z) &\equiv \delta_{\bar{a}b} \psi^{\bar{a}i} t_{i\bar{j}}^\rho \psi^{b\bar{j}}(z), \\
j^{u(1)}(z) &\equiv \delta_{\bar{a}b} \delta_{i\bar{j}} \psi^{a\bar{i}} \psi^{b\bar{j}}(z).
\end{aligned} \tag{C.3} \{?\}$$

Note that $\hat{J}^{u(1)}(z) \equiv \sqrt{\frac{NM}{(N+M)}} J^{u(1)}(z)$. Then the currents in the denominator of the coset (1.1) are given by $(J^\alpha + j^\alpha)(z)$ for $SU(N)$ factor, $(J^\rho + j^\rho)(z)$ for $SU(M)$ factor, and $(N+M)(\hat{J}^{u(1)} + j^{u(1)})(z)$ for $U(1)$ factor where $J^\alpha(z) \equiv (J_1^\alpha + J_2^\alpha)(z)$ and $J^\rho(z) \equiv (J_1^\rho + J_2^\rho)(z)$.

C.3 The OPEs between the spin-1 currents and itself

Furthermore, one obtains the following OPEs by expanding the structure constants explicitly as before where the spin-1 currents are given in Appendix (C.3)

$$\begin{aligned}
J_1^\alpha(z) J_1^\beta(w) &= \frac{1}{(z-w)^2} N \delta^{\alpha\beta} + \frac{1}{(z-w)} i f^{\alpha\beta\gamma} J_1^\gamma(w) + \dots, \\
J_1^\alpha(z) J^{\bar{a}\bar{i}}(w) &= \frac{1}{(z-w)} i f^{\alpha\beta\gamma} \Psi^\gamma \Psi^{\bar{b}\bar{i}} t_{b\bar{a}}^\beta(w) + \dots, \\
J_1^\alpha(z) J^{\bar{a}i}(w) &= -\frac{1}{(z-w)} i f^{\alpha\beta\gamma} \Psi^\gamma \Psi^{\bar{b}i} t_{a\bar{b}}^\beta(w) + \dots, \\
J_2^\alpha(z) J_2^\beta(w) &= \frac{1}{(z-w)^2} M \delta^{\alpha\beta} + \frac{1}{(z-w)} i f^{\alpha\beta\gamma} J_2^\gamma(w) + \dots, \\
J_2^\alpha(z) J^{\bar{a}\bar{i}}(w) &= \frac{1}{(z-w)} \left(t_{b\bar{a}}^\alpha J^{\bar{b}\bar{i}} - i f^{\alpha\beta\gamma} \Psi^\gamma \Psi^{\bar{b}\bar{i}} t_{b\bar{a}}^\beta \right)(w) + \dots, \\
J_2^\alpha(z) J^{\bar{a}i}(w) &= \frac{1}{(z-w)} \left(-J^{\bar{b}i} t_{a\bar{b}}^\alpha + i f^{\alpha\beta\gamma} \Psi^\gamma t_{a\bar{b}}^\beta \Psi^{\bar{b}i} \right)(w) + \dots, \\
J_1^\rho(z) J^{\bar{a}\bar{i}}(w) &= -\frac{1}{(z-w)} i f^{\rho\sigma\tau} \Psi^\tau t_{i\bar{j}}^\sigma \Psi^{a\bar{j}}(w) + \dots, \\
J_1^\rho(z) J^{\bar{a}i}(w) &= \frac{1}{(z-w)} i f^{\rho\sigma\tau} \Psi^\tau \Psi^{\bar{a}j} t_{j\bar{i}}^\sigma(w) + \dots, \\
J_1^\rho(z) J_1^\sigma(w) &= \frac{1}{(z-w)^2} M \delta^{\rho\sigma} + \frac{1}{(z-w)} i f^{\rho\sigma\tau} J_1^\tau(w) + \dots, \\
J_2^\rho(z) J_2^\sigma(w) &= \frac{1}{(z-w)^2} N \delta^{\rho\sigma} + \frac{1}{(z-w)} i f^{\rho\sigma\tau} J_2^\tau(w) + \dots, \\
J_2^\rho(z) J^{\bar{a}\bar{i}}(w) &= \frac{1}{(z-w)} \left(-J^{\bar{a}\bar{j}} t_{i\bar{j}}^\rho + i f^{\rho\sigma\tau} \Psi^\tau t_{i\bar{j}}^\sigma \Psi^{\bar{a}\bar{j}} \right)(w) + \dots, \\
J_2^\rho(z) J^{\bar{a}i}(w) &= \frac{1}{(z-w)} \left(J^{\bar{a}j} t_{j\bar{i}}^\rho - i f^{\rho\sigma\tau} \Psi^\tau \Psi^{\bar{a}j} t_{j\bar{i}}^\sigma \right)(w) + \dots, \\
J^{u(1)}(z) J^{u(1)}(w) &= \frac{1}{(z-w)^2} (N+M) + \dots, \\
J^{u(1)}(z) J^{\bar{a}\bar{i}}(w) &= \frac{1}{(z-w)} \sqrt{\frac{N+M}{NM}} J^{\bar{a}\bar{i}}(w) + \dots, \\
J^{u(1)}(z) J^{\bar{a}i}(w) &= -\frac{1}{(z-w)} \sqrt{\frac{N+M}{NM}} J^{\bar{a}i}(w) + \dots,
\end{aligned}$$

$$\begin{aligned}
J^{a\bar{i}}(z) J^{\bar{b}j}(w) &= \frac{1}{(z-w)^2} (N+M) \delta^{a\bar{b}} \delta^{\bar{i}j} \\
&+ \frac{1}{(z-w)} \left(\delta^{\bar{i}j} t_{b\bar{a}}^\alpha (J_1^\alpha + J_2^\alpha) - \delta^{a\bar{b}} t_{i\bar{j}}^\rho (J_1^\rho + J_2^\rho) + \sqrt{\frac{N+M}{NM}} \delta^{a\bar{b}} \delta^{\bar{i}j} J^{u(1)} \right) (w) \\
&+ \dots, \\
j^\alpha(z) j^\beta(w) &= \frac{1}{(z-w)^2} M \delta^{\alpha\beta} + \frac{1}{(z-w)} i f^{\alpha\beta\gamma} j^\gamma(w) + \dots, \\
j^\rho(z) j^\sigma(w) &= \frac{1}{(z-w)^2} N \delta^{\rho\sigma} + \frac{1}{(z-w)} i f^{\rho\sigma\tau} j^\tau(w) + \dots, \\
j^{u(1)}(z) j^{u(1)}(w) &= \frac{1}{(z-w)^2} NM + \dots,
\end{aligned} \tag{C.4} \{?\}$$

corresponding to (3.21) of [18]. We observe that the levels are given by $(N+2M)$, $(2N+M)$ and $2NM(N+M)^2$ corresponding to the $SU(N)$, $SU(M)$ and $U(1)$ factors appearing on the denominator of the coset (1.1) respectively. This can be seen from the second-order pole of each OPE between the corresponding current we described in previous subsection. The first level is obtained from the first, the fourth and the seventeenth equations of (C.4). The second level is given by the ninth, the tenth and the eighteenth equations of (C.4). Finally the third level can be obtained from the thirteenth and the last equations of (C.4).

D The remaining higher spin currents in section 3

Let us introduce the following (N, M) dependent coefficient functions

$$\begin{aligned}
A(N, M) &\equiv \sqrt{\frac{3N^2M}{2(M+N)(2M+3N)(MN-1)}}, \\
B(N, M) &\equiv A(M, N) = \sqrt{\frac{3M^2N}{2(M+N)(2N+3M)(MN-1)}}, \\
a(N, M) &\equiv -\frac{2M}{3N}, \\
b(N, M) &\equiv a(M, N) = -\frac{2N}{3M}.
\end{aligned} \tag{D.1} \{?\}$$

We present the remaining 7 higher spin currents in terms of various fermions as follows with the help of (C.1), (C.2) and (C.4).

D.1 The higher spin-2 currents

By using (3.12) and (3.13), the three higher spin-2 currents, together with (D.1), can be described as

$$\phi^{(2),+}(z) = -\frac{A(N, M)}{\sqrt{N+M}} t_{b\bar{a}}^\alpha (\psi^{\bar{a}i} \Psi^{\bar{b}i}) \left((3a(N, M) - 2) J_1^\alpha + J_2^\alpha + j^\alpha \right) (z)$$

$$\begin{aligned}
& + \frac{B(N, M)}{\sqrt{N+M}} t_{ij}^\rho (\psi^{\bar{a}i} \Psi^{a\bar{j}}) \left((3b(N, M) - 2) J_1^\rho + J_2^\rho + j^\rho \right) (z), \\
\phi^{(2),-}(z) & = \frac{A(N, M)}{\sqrt{N+M}} t_{ab}^\alpha (\psi^{a\bar{i}} \Psi^{\bar{b}i}) \left((3a(N, M) - 2) J_1^\alpha + J_2^\alpha + j^\alpha \right) (z) \\
& - \frac{B(N, M)}{\sqrt{N+M}} t_{ji}^\rho (\psi^{a\bar{i}} \Psi^{\bar{a}j}) \left((3b(N, M) - 2) J_1^\rho + J_2^\rho + j^\rho \right) (z), \\
\phi^{(2),3}(z) & = \frac{A(N, M)}{\sqrt{2(N+M)}} (J_2^\alpha - j^\alpha) \left((3a(N, M) - 2) J_1^\alpha + J_2^\alpha + j^\alpha \right) (z) \\
& + \frac{B(N, M)}{\sqrt{2(N+M)}} (J_2^\rho - j^\rho) \left((3b(N, M) - 2) J_1^\rho + J_2^\rho + j^\rho \right) (z). \tag{D.2} \{?\}
\end{aligned}$$

The first two of (D.2) are not fully normal ordered products in the spirit of [72, 73, 1].

D.2 The higher spin- $\frac{5}{2}$ currents

The three higher spin- $\frac{5}{2}$ currents by using (3.16) are summarized by

$$\begin{aligned}
\psi^{(\frac{5}{2}),+}(z) & = \frac{A(N, M)}{\sqrt{2(N+M)}} \left[2t_{ba}^\alpha (\psi^{\bar{a}i} J^{\bar{b}i}) \left((3a(N, M) - 2) J_1^\alpha + J_2^\alpha + j^\alpha \right) + i f^{\alpha\beta\gamma} (\psi^{\bar{a}i} \Psi^\beta \Psi^{\bar{b}i}) \right. \\
& \times \left. \left((3a(N, M) - 2) J_1^\alpha + (4 - 3a(N, M)) J_2^\alpha + (3a(N, M) - 2) j^\alpha \right) t_{ba}^\gamma \right] (z) \\
& - \frac{B(N, M)}{\sqrt{2(N+M)}} \left[2t_{ij}^\rho (\psi^{\bar{a}i} J^{a\bar{j}}) \left((3b(N, M) - 2) J_1^\rho + J_2^\rho + j^\rho \right) + i f^{\rho\sigma\tau} (\psi^{\bar{a}i} \Psi^\sigma \Psi^{a\bar{j}}) \right. \\
& \times \left. \left((3b(N, M) - 2) J_1^\rho + (4 - 3b(N, M)) J_2^\rho + (3b(N, M) - 2) j^\rho \right) t_{ij}^\tau \right] (z), \\
\psi^{(\frac{5}{2}),-}(z) & = \frac{A(N, M)}{\sqrt{2(N+M)}} \left[2t_{ab}^\alpha (\psi^{a\bar{i}} J^{\bar{b}i}) \left((3a(N, M) - 2) J_1^\alpha + J_2^\alpha + j^\alpha \right) + i f^{\alpha\beta\gamma} (\psi^{a\bar{i}} \Psi^\beta \Psi^{\bar{b}i}) \right. \\
& \times \left. \left((3a(N, M) - 2) J_1^\alpha + (4 - 3a(N, M)) J_2^\alpha + (3a(N, M) - 2) j^\alpha \right) t_{ab}^\gamma \right] (z) \\
& - \frac{B(N, M)}{\sqrt{2(N+M)}} \left[2t_{ji}^\rho (\psi^{a\bar{i}} J^{\bar{a}j}) \left((3b(N, M) - 2) J_1^\rho + J_2^\rho + j^\rho \right) + i f^{\rho\sigma\tau} (\psi^{a\bar{i}} \Psi^\sigma \Psi^{\bar{a}j}) \right. \\
& \times \left. \left((3b(N, M) - 2) J_1^\rho + (4 - 3b(N, M)) J_2^\rho + (3b(N, M) - 2) j^\rho \right) t_{ji}^\tau \right] (z), \\
\psi^{(\frac{5}{2}),3}(z) & = \frac{A(N, M)}{(N+M)} \left[t_{ab}^\alpha \left(J^{a\bar{i}} \Psi^{\bar{b}i} - \sqrt{\frac{N+M}{NM}} \psi^{a\bar{i}} \psi^{\bar{b}i} \Psi^{u(1)} - \psi^{a\bar{i}} \psi^{\bar{c}i} \Psi^\beta t_{b\bar{c}}^\beta + \psi^{a\bar{i}} \psi^{\bar{b}j} \Psi^\rho t_{j\bar{i}}^\rho \right) \right. \\
& \times \left. \left((3a(N, M) - 2) J_1^\alpha + J_2^\alpha + j^\alpha \right) + 3i(1 - a(N, M)) f^{\alpha\beta\gamma} t_{ab}^\alpha t_{d\bar{c}}^\beta (\psi^{a\bar{i}} \Psi^{\bar{b}i}) (\psi^{\bar{c}j} \Psi^\gamma \Psi^{d\bar{j}}) \right] (z) \\
& + \frac{B(N, M)}{(N+M)} \left[t_{ji}^\rho \left(-J^{a\bar{i}} \Psi^{\bar{a}j} + \sqrt{\frac{N+M}{NM}} \psi^{a\bar{i}} \psi^{\bar{a}j} \Psi^{u(1)} + \psi^{a\bar{i}} \psi^{\bar{b}j} \Psi^\beta t_{a\bar{b}}^\beta - \psi^{a\bar{i}} \psi^{\bar{a}k} \Psi^\sigma t_{k\bar{j}}^\sigma \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \times \left((3b(N, M) - 2)J_1^\rho + J_2^\rho + j^\rho \right) + 3i(1 - b(N, M))f^{\rho\sigma\tau}t_{j\bar{i}}^\rho t_{k\bar{l}}^\sigma (\psi^{a\bar{i}}\Psi^{\bar{a}j})(\psi^{\bar{b}k}\Psi^\tau\Psi^{\bar{l}}) \Big] (z) \\
& - \partial\psi^{(\frac{3}{2})}(z), \tag{D.3} \{?\}
\end{aligned}$$

with (D.1). The lowest higher spin- $\frac{3}{2}$ current appearing in Appendix (D.3) is given by (3.11). In order to express the above in the form of fully normal ordered products we should further arrange them carefully.

D.3 The higher spin-3 current

By using (3.17) and (3.18), the higher spin-3 current, together with (D.1), can be written as

$$\begin{aligned}
\phi^{(3)}(z) &= \frac{A(N, M)}{\sqrt{8(N+M)^3}} \left[2t_{ab}^\alpha \left(J^{a\bar{i}}J^{\bar{b}i} - \psi^{a\bar{i}}\psi^{\bar{c}i}t_{b\bar{c}}^\beta (J_1^\beta + J_2^\beta) + \psi^{a\bar{i}}\psi^{\bar{b}j}t_{j\bar{i}}^\rho (J_1^\rho + J_2^\rho) \right. \right. \\
&- \sqrt{\frac{(N+M)}{NM}}\psi^{a\bar{i}}\psi^{\bar{b}i}J^{u(1)} - (N+M)\psi^{a\bar{i}}\partial\psi^{\bar{b}i} \Big) ((3a(N, M) - 2)J_1^\alpha + J_2^\alpha + j^\alpha) \\
&+ 6i(a(N, M) - 1)f^{\alpha\beta\gamma}t_{ab}^\alpha t_{d\bar{c}}^\beta (\psi^{a\bar{i}}J^{\bar{b}i})(\psi^{\bar{c}j}\Psi^\gamma\Psi^{d\bar{j}}) \\
&+ if^{\alpha\beta\gamma} \left(J^{a\bar{i}}\Psi^\beta\Psi^{\bar{b}i} + \psi^{a\bar{i}}(\psi^{\bar{c}j}\Psi^{d\bar{j}})\Psi^{\bar{b}i}t_{d\bar{c}}^\beta - \sqrt{\frac{(N+M)}{NM}}\psi^{a\bar{i}}\psi^{\bar{b}i}\Psi^\beta\Psi^{u(1)} \right. \\
&- \psi^{a\bar{i}}\psi^{\bar{c}i}\Psi^\beta\Psi^\delta t_{b\bar{c}}^\delta + \psi^{a\bar{i}}\psi^{\bar{b}j}\Psi^\beta\Psi^\rho t_{j\bar{i}}^\rho \Big) \left((3a(N, M) - 2)J_1^\alpha + (4 - 3a(N, M))J_2^\alpha \right. \\
&+ (3a(N, M) - 2)j^\alpha \Big) t_{ab}^\gamma + 6i(1 - a(N, M))f^{\alpha\beta\gamma}(\psi^{a\bar{i}}\Psi^\beta\Psi^{\bar{b}i}) \\
&\times \left(if^{\alpha\delta\epsilon}\psi^{\bar{c}j}\Psi^\delta\Psi^{d\bar{j}}t_{d\bar{c}}^\epsilon + J^{c\bar{j}}\psi^{\bar{d}j}t_{cd}^\alpha \right) t_{ab}^\gamma \Big] (z) \\
&+ \frac{B(N, M)}{\sqrt{8(N+M)^3}} \left[2t_{j\bar{i}}^\rho \left(-J^{a\bar{i}}J^{\bar{a}j} + \psi^{a\bar{i}}\psi^{\bar{b}j}t_{a\bar{b}}^\alpha (J_1^\alpha + J_2^\alpha) - \psi^{a\bar{i}}\psi^{\bar{a}k}t_{k\bar{j}}^\sigma (J_1^\sigma + J_2^\sigma) \right. \right. \\
&+ \sqrt{\frac{N+M}{NM}}\psi^{a\bar{i}}\psi^{\bar{a}j}J^{u(1)} + (N+M)\psi^{a\bar{i}}\partial\psi^{\bar{a}j} \Big) \left((3b(N, M) - 2)J_1^\rho + J_2^\rho + j^\rho \right) \\
&+ 6i(b(N, M) - 1)f^{\rho\sigma\tau}t_{j\bar{i}}^\rho t_{k\bar{l}}^\sigma (\psi^{a\bar{i}}J^{\bar{a}j})(\psi^{\bar{b}k}\Psi^\tau\Psi^{\bar{l}}) \\
&- if^{\rho\sigma\tau} \left(J^{a\bar{i}}\Psi^\sigma\Psi^{\bar{a}j} - \psi^{a\bar{i}}(\psi^{\bar{b}k}\Psi^{b\bar{l}})\Psi^{\bar{a}j}t_{k\bar{l}}^\sigma - \sqrt{\frac{N+M}{NM}}\psi^{a\bar{i}}\psi^{\bar{a}j}\Psi^\sigma\Psi^{u(1)} \right. \\
&- \psi^{a\bar{i}}\psi^{\bar{b}j}\Psi^\sigma\Psi^\gamma t_{ab}^\gamma + \psi^{a\bar{i}}\psi^{\bar{a}k}\Psi^\sigma\Psi^\nu t_{k\bar{j}}^\nu \Big) \\
&\times \left((3b(N, M) - 2)J_1^\rho + (4 - 3b(N, M))J_2^\rho + (3b(N, M) - 2)j^\rho \right) t_{j\bar{i}}^\tau \\
&+ 6i(1 - b(N, M))f^{\rho\sigma\tau}(\psi^{a\bar{i}}\Psi^\sigma\Psi^{\bar{a}j}) \left(if^{\rho\nu\phi}\psi^{\bar{b}k}\Psi^\nu\Psi^{b\bar{l}}t_{k\bar{l}}^\phi + \psi^{\bar{b}k}J^{\bar{b}l}t_{k\bar{l}}^\rho \right) t_{j\bar{i}}^\tau \Big] (z) \\
&- \frac{1}{2}\partial\phi^{(2),3}(z), \tag{D.4} \{?\}
\end{aligned}$$

where one of the higher spin-2 current appearing in Appendix (D.4) is given by Appendix (D.2). Further rearrangement of the multiple products should be done in order to write this in terms of fully normal ordered product [72, 73, 1].

E The OPEs in the $\mathcal{N} = 2$ superspace in section 6

In this Appendix, we present the remaining $\mathcal{N} = 2$ OPEs discussed in section 6.

E.1 The OPEs in $\mathcal{N} = 2$ superspace corresponding to Appendix A

In order to describe the $\mathcal{N} = 2$ description for the $\mathcal{N} = 2$ OPEs between the higher spin currents, we should write down the standard $\mathcal{N} = 3$ superconformal algebra in $\mathcal{N} = 2$ superspace. By writing down each component current with an appropriate combinations correctly, the following $\mathcal{N} = 2$ stress energy tensor satisfies the standard $\mathcal{N} = 2$ OPE

$$\begin{aligned}\mathbf{T}(Z) &= J^3(z) - \theta \frac{1}{2}(G^1 + iG^2)(z) + \bar{\theta} \frac{1}{2}(-G^1 + iG^2)(z) + \theta\bar{\theta} T(z) \\ &\equiv \left(J^3, \quad -\frac{1}{2}(G^1 + iG^2), \quad \frac{1}{2}(-G^1 + iG^2), \quad T \right).\end{aligned}\tag{E.1} \{?\}$$

Then the remaining four currents of $\mathcal{N} = 3$ superconformal algebra can combine as the following $\mathcal{N} = 2$ multiplet

$$\begin{aligned}\mathbf{T}^{(\frac{1}{2})}(Z) &= 2\Psi(z) + \theta(J^1 + iJ^2)(z) + \bar{\theta}(-J^1 + iJ^2)(z) + \theta\bar{\theta} G^3(z) \\ &\equiv \left(2\Psi, \quad (J^1 + iJ^2), \quad (-J^1 + iJ^2), \quad G^3 \right).\end{aligned}\tag{E.2} \{?\}$$

Then we identify the previous component results presented in Appendix (A.1) can be interpreted as the following three $\mathcal{N} = 2$ OPEs between the $\mathcal{N} = 2$ stress energy tensor (E.1) and the $\mathcal{N} = 2$ multiplet (E.2)

$$\begin{aligned}\mathbf{T}(Z_1) \mathbf{T}(Z_2) &= \frac{1}{z_{12}^2} \frac{c}{3} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} \mathbf{T}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D}\mathbf{T}(Z_2) - \frac{\theta_{12}}{z_{12}} D\mathbf{T}(Z_2) \\ &\quad + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial\mathbf{T}(Z_2) + \dots, \\ \mathbf{T}(Z_1) \mathbf{T}^{(\frac{1}{2})}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} \frac{1}{2} \mathbf{T}^{(\frac{1}{2})}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D}\mathbf{T}^{(\frac{1}{2})}(Z_2) - \frac{\theta_{12}}{z_{12}} D\mathbf{T}^{(\frac{1}{2})}(Z_2) \\ &\quad + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial\mathbf{T}^{(\frac{1}{2})}(Z_2) + \dots, \\ \mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{T}^{(\frac{1}{2})}(Z_2) &= \frac{1}{z_{12}} \frac{4c}{3} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} 2\mathbf{T}(Z_2) + \dots.\end{aligned}\tag{E.3} \{?\}$$

Note that this $\mathcal{N} = 2$ OPEs (E.3) can be obtained from the large $\mathcal{N} = 4$ superconformal algebra observed in [26, 74]. The complex spinor covariant derivatives are given by $D = \frac{\partial}{\partial\theta} - \frac{1}{2}\bar{\theta}\frac{\partial}{\partial z}$ and $\bar{D} = \frac{\partial}{\partial\bar{\theta}} - \frac{1}{2}\theta\frac{\partial}{\partial z}$ [30].

E.2 The OPEs in $\mathcal{N} = 2$ superspace corresponding to Appendix B

Furthermore, the component results presented in Appendix B can be described in $\mathcal{N} = 2$ superspace as follows:

$$\begin{aligned}
\mathbf{T}(Z_1) \mathbf{W}^{(\frac{3}{2})}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} \frac{3}{2} \mathbf{W}^{(\frac{3}{2})}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(\frac{3}{2})}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(\frac{3}{2})}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(\frac{3}{2})}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(2)}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 2 \mathbf{W}^{(2)}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(2)}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(2)}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(2)}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(2')}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 2 \mathbf{W}^{(2')}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(2')}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(2')}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(2')}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(\frac{5}{2})}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} \frac{5}{2} \mathbf{W}^{(\frac{5}{2})}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(\frac{5}{2})}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(\frac{5}{2})}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(\frac{5}{2})}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(\frac{5}{2})'}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} \frac{5}{2} \mathbf{W}^{(\frac{5}{2})'}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(\frac{5}{2})'}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(\frac{5}{2})'}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(\frac{5}{2})'}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(3)}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 3 \mathbf{W}^{(3)}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(3)}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(3)}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(3)}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(3),1}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 3 \mathbf{W}^{(3),1}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(3),1}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(3),1}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(3),1}(Z_2) + \frac{1}{z_{12}} i \mathbf{W}^{(3),2}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(\frac{7}{2}),1}(Z_2) &= \frac{\theta_{12}}{z_{12}^2} \frac{1}{2} \mathbf{W}^{(3),3}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}^2} \frac{1}{2} \mathbf{W}^{(3),3}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} \frac{7}{2} \mathbf{W}^{(\frac{7}{2}),1}(Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(\frac{7}{2}),1}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(\frac{7}{2}),1}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(\frac{7}{2}),1}(Z_2) \\
&+ \frac{1}{z_{12}} i \mathbf{W}^{(\frac{7}{2}),2}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(3),2}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 3 \mathbf{W}^{(3),2}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(3),2}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(3),2}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(3),2}(Z_2) - \frac{1}{z_{12}} i \mathbf{W}^{(3),1}(Z_2) + \dots,
\end{aligned}$$

$$\begin{aligned}
\mathbf{T}(Z_1) \mathbf{W}^{(\frac{7}{2}),2}(Z_2) &= \frac{\theta_{12}}{z_{12}^2} \frac{i}{2} \mathbf{W}^{(3),3}(Z_2) - \frac{\bar{\theta}_{12}}{z_{12}^2} \frac{i}{2} \mathbf{W}^{(3),3}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} \frac{7}{2} \mathbf{W}^{(\frac{7}{2}),2}(Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(\frac{7}{2}),2}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(\frac{7}{2}),2}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(\frac{7}{2}),2}(Z_2) \\
&- \frac{1}{z_{12}} i \mathbf{W}^{(\frac{7}{2}),1}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(3),3}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 3 \mathbf{W}^{(3),3}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(3),3}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(3),3}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(3),3}(Z_2) + \dots, \\
\mathbf{T}(Z_1) \mathbf{W}^{(\frac{7}{2}),3}(Z_2) &= -\frac{\theta_{12}}{z_{12}^2} \frac{1}{2} \mathbf{W}^{(3),1}(Z_2) - \frac{\bar{\theta}_{12}}{z_{12}^2} \frac{1}{2} \mathbf{W}^{(3),1}(Z_2) - \frac{\theta_{12}}{z_{12}^2} \frac{i}{2} \mathbf{W}^{(3),2}(Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}^2} \frac{i}{2} \mathbf{W}^{(3),2}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} \frac{7}{2} \mathbf{W}^{(\frac{7}{2}),3}(Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}} \bar{D} \mathbf{W}^{(\frac{7}{2}),3}(Z_2) - \frac{\theta_{12}}{z_{12}} D \mathbf{W}^{(\frac{7}{2}),3}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(\frac{7}{2}),3}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(\frac{3}{2})}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \mathbf{W}^{(2)}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(2)}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 3 \mathbf{W}^{(\frac{3}{2})}(Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}} 2 \bar{D} \mathbf{W}^{(\frac{3}{2})}(Z_2) - \frac{\theta_{12}}{z_{12}} 2 D \mathbf{W}^{(\frac{3}{2})}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(\frac{3}{2})}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(2')}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \mathbf{W}^{(\frac{5}{2})}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(\frac{5}{2})}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 4 \mathbf{W}^{(2')}(Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}} 2 \bar{D} \mathbf{W}^{(2')}(Z_2) - \frac{\theta_{12}}{z_{12}} 2 D \mathbf{W}^{(2')}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(2')}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(\frac{5}{2}')} (Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \mathbf{W}^{(3)}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(3)}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 5 \mathbf{W}^{(\frac{5}{2}')} (Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}} 2 \bar{D} \mathbf{W}^{(\frac{5}{2}')} (Z_2) - \frac{\theta_{12}}{z_{12}} 2 D \mathbf{W}^{(\frac{5}{2}')} (Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(\frac{5}{2}')} (Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(3),1}(Z_2) &= \frac{\bar{\theta}_{12}}{z_{12}} \mathbf{W}^{(3),3}(Z_2) + \frac{\theta_{12}}{z_{12}} \mathbf{W}^{(3),3}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \mathbf{W}^{(\frac{7}{2}),1}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(\frac{7}{2}),1}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 6 \mathbf{W}^{(3),1}(Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}} \left[2 \bar{D} \mathbf{W}^{(3),1} + \mathbf{W}^{(\frac{7}{2}),3} \right] (Z_2) + \frac{\theta_{12}}{z_{12}} \left[-2 D \mathbf{W}^{(3),1} + \mathbf{W}^{(\frac{7}{2}),3} \right] (Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial \mathbf{W}^{(3),1}(Z_2) + \frac{1}{z_{12}} 2i \mathbf{W}^{(3),2}(Z_2) + \dots,
\end{aligned}$$

$$\begin{aligned}
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(3),2}(Z_2) &= -\frac{\bar{\theta}_{12}}{z_{12}} i\mathbf{W}^{(3),3}(Z_2) + \frac{\theta_{12}}{z_{12}} i\mathbf{W}^{(3),3}(Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \mathbf{W}^{(\frac{7}{2}),2}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(\frac{7}{2}),2}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 6\mathbf{W}^{(3),2}(Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}} \left[2\bar{D}\mathbf{W}^{(3),2} - i\mathbf{W}^{(\frac{7}{2}),3} \right] (Z_2) + \frac{\theta_{12}}{z_{12}} \left[-2D\mathbf{W}^{(3),2} + i\mathbf{W}^{(\frac{7}{2}),3} \right] (Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial\mathbf{W}^{(3),2}(Z_2) - \frac{1}{z_{12}} 2i\mathbf{W}^{(3),1}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(3),3}(Z_2) &= \frac{\bar{\theta}_{12}}{z_{12}} \left[-\mathbf{W}^{(3),1} + i\mathbf{W}^{(3),2} \right] (Z_2) - \frac{\theta_{12}}{z_{12}} \left[\mathbf{W}^{(3),1} + i\mathbf{W}^{(3),2} \right] (Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \mathbf{W}^{(\frac{7}{2}),3}(Z_2) + \dots, \\
\mathbf{T}^{(\frac{1}{2})}(Z_1) \mathbf{W}^{(\frac{7}{2}),3}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} 6\mathbf{W}^{(3),3}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \left[2\bar{D}\mathbf{W}^{(3),3} - \mathbf{W}^{(\frac{7}{2}),1} + i\mathbf{W}^{(\frac{7}{2}),2} \right] (Z_2) \\
&- \frac{\theta_{12}}{z_{12}} \left[2D\mathbf{W}^{(3),3} + \mathbf{W}^{(\frac{7}{2}),1} + i\mathbf{W}^{(\frac{7}{2}),2} \right] (Z_2) + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \partial\mathbf{W}^{(3),3}(Z_2) + \dots.
\end{aligned} \tag{E.4} \{?\}$$

We see that the six higher spin currents, $\mathbf{W}^{(3),\alpha}(Z_2)$ and $\mathbf{W}^{(\frac{7}{2}),\alpha}(Z_2)$ are not primary under the $\mathcal{N} = 2$ stress energy tensor $\mathbf{T}(Z_1)$. We observe that the combination of $(\mathbf{W}^{(3),1} \pm i\mathbf{W}^{(3),2})(Z_2)$ has $U(1)$ charge ± 1 by calculating the OPE with $\mathbf{T}(Z_1)$

$$\mathbf{T}(Z_1) (\mathbf{W}^{(3),1} \pm i\mathbf{W}^{(3),2})(Z_2) = \pm \frac{1}{z_{12}} (\mathbf{W}^{(3),1} \pm i\mathbf{W}^{(3),2})(Z_2) + \text{sing. terms.} \tag{E.5} \{?\}$$

Similarly, we can see

$$\mathbf{T}(Z_1) (\mathbf{W}^{(\frac{7}{2}),1} \pm i\mathbf{W}^{(\frac{7}{2}),2})(Z_2) = \pm \frac{1}{z_{12}} (\mathbf{W}^{(\frac{7}{2}),1} \pm i\mathbf{W}^{(\frac{7}{2}),2})(Z_2) + \text{sing. terms.} \tag{E.6} \{?\}$$

Therefore, $(\mathbf{W}^{(\frac{7}{2}),1} \pm i\mathbf{W}^{(\frac{7}{2}),2})(Z_2)$ have the $U(1)$ charges (of $\mathcal{N} = 2$ superconformal algebra) ± 1 respectively.

E.3 The remaining two OPEs in $\mathcal{N} = 2$ superspace

The $\mathcal{N} = 2$ OPE between the $\mathcal{N} = 2$ higher spin- $\frac{3}{2}$ current and the $\mathcal{N} = 2$ higher spin-2 current can be summarized by

$$\begin{aligned}
\mathbf{W}^{(\frac{3}{2})}(Z_1) \mathbf{W}^{(2)}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^4} 6\mathbf{T}^{(\frac{1}{2})}(Z_2) - \frac{\theta_{12}}{z_{12}^3} 12D\mathbf{T}^{(\frac{1}{2})}(Z_2) + \frac{\bar{\theta}_{12}}{z_{12}^3} 12\bar{D}\mathbf{T}^{(\frac{1}{2})}(Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^3} 6\partial\mathbf{T}^{(\frac{1}{2})}(Z_2) \\
&+ \frac{1}{z_{12}^2} \frac{1}{(2c-3)} \left[-36\mathbf{T}\mathbf{T}^{(\frac{1}{2})} - 12c[D, \bar{D}]\mathbf{T}^{(\frac{1}{2})} \right] (Z_2) \\
&+ \frac{\theta_{12}}{z_{12}^2} \frac{1}{(2c-3)} \left[-12(c-3)\partial D\mathbf{T}^{(\frac{1}{2})} - 36\mathbf{T}D\mathbf{T}^{(\frac{1}{2})} \right] (Z_2)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\bar{\theta}_{12}}{z_{12}^2} \frac{1}{(2c-3)} \left[12(c-3) \partial \bar{D} \mathbf{T}^{(\frac{1}{2})} - 36 \mathbf{T} \bar{D} \mathbf{T}^{(\frac{1}{2})} \right] (Z_2) \\
& + \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^2} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-18(13c+18) \mathbf{T} \mathbf{T} \mathbf{T}^{(\frac{1}{2})} \right. \right. \\
& + 3(2c^3 - 25c^2 - 123c - 126) \partial^2 \mathbf{T}^{(\frac{1}{2})} + 144c(c+1) D \mathbf{T} \bar{D} \mathbf{T}^{(\frac{1}{2})} \\
& - 144c(c+1) \bar{D} \mathbf{T} D \mathbf{T}^{(\frac{1}{2})} + 18(13c+18) \mathbf{T}^{(\frac{1}{2})} \bar{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \\
& - 12(2c+3)(c+6) [D, \bar{D}] \mathbf{T} \mathbf{T}^{(\frac{1}{2})} - 54(c-2)(c+1) \mathbf{T} [D, \bar{D}] \mathbf{T}^{(\frac{1}{2})} \\
& \left. \left. - 18(13c+18) \partial \mathbf{T} \mathbf{T}^{(\frac{1}{2})} \right) + \frac{3}{2c} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \mathbf{T}^{(\frac{1}{2})} \mathbf{W}^{(2')} + 2C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \mathbf{W}^{(\frac{5}{2}')} \right] (Z_2) \\
& + \frac{1}{z_{12}} \left[\frac{1}{(c+1)(2c-3)} \left(-12(c+3) \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} - 12c \partial \mathbf{T} \mathbf{T}^{(\frac{1}{2})} \right. \right. \\
& - 4(c-3)c \partial [D, \bar{D}] \mathbf{T}^{(\frac{1}{2})} + 24c \bar{D} \mathbf{T} D \mathbf{T}^{(\frac{1}{2})} + 24c D \mathbf{T} \bar{D} \mathbf{T}^{(\frac{1}{2})} \left. \left. \right) - C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \mathbf{W}^{(\frac{5}{2})} \right] (Z_2) \\
& + \frac{\theta_{12}}{z_{12}} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-12(c^2 + 24c + 36) \mathbf{T} \partial D \mathbf{T}^{(\frac{1}{2})} \right. \right. \\
& - 12(2c^2 - 27c - 54) \partial \mathbf{T} D \mathbf{T}^{(\frac{1}{2})} + 36(5c+6) D \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} \\
& + 36c \partial \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} - 2(2c^3 - 27c^2 - 198c - 216) \partial^2 D \mathbf{T}^{(\frac{1}{2})} \\
& + 12(4c^2 + 15c + 18) [D, \bar{D}] \mathbf{T} D \mathbf{T}^{(\frac{1}{2})} - 108(c+2) \bar{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \\
& + 12c(5c+6) D \mathbf{T} [D, \bar{D}] \mathbf{T}^{(\frac{1}{2})} + 9(5c+6) \mathbf{T}^{(\frac{1}{2})} [D, \bar{D}] \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \\
& \left. \left. + 108(c+2) \mathbf{T} \mathbf{T} D \mathbf{T}^{(\frac{1}{2})} + 36(5c+6) \mathbf{T} D \mathbf{T} \mathbf{T}^{(\frac{1}{2})} \right) \right. \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{3(c+3)}{5(c-3)c} \mathbf{T}^{(\frac{1}{2})} D \mathbf{W}^{(2')} - \frac{9(3c-1)}{5(c-3)c} D \mathbf{T}^{(\frac{1}{2})} \mathbf{W}^{(2')} \right. \\
& \left. \left. - \frac{2(c+3)}{5(c-3)} D \mathbf{W}^{(\frac{5}{2})} \right) - \frac{4}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} D \mathbf{W}^{(\frac{5}{2}')} - C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(\mathbf{W}^{(3),1} + i \mathbf{W}^{(3),2} \right) \right] (Z_2) \\
& + \frac{\bar{\theta}_{12}}{z_{12}} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-12(c^2 + 24c + 36) \mathbf{T} \partial \bar{D} \mathbf{T}^{(\frac{1}{2})} \right. \right. \\
& - 12(2c^2 + 9c + 18) \partial \mathbf{T} \bar{D} \mathbf{T}^{(\frac{1}{2})} + 36(5c+6) \bar{D} \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} \\
& - 36(5c+6) \partial \bar{D} \mathbf{T} \mathbf{T}^{(\frac{1}{2})} - 36c \partial \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} \bar{D} \mathbf{T}^{(\frac{1}{2})} \\
& + 2(2c^3 - 27c^2 - 207c - 270) \partial^2 \bar{D} \mathbf{T}^{(\frac{1}{2})} \\
& - 12(4c^2 + 15c + 18) [D, \bar{D}] \mathbf{T} \bar{D} \mathbf{T}^{(\frac{1}{2})} - 12c(5c+6) \bar{D} \mathbf{T} [D, \bar{D}] \mathbf{T}^{(\frac{1}{2})} \\
& + 9(5c+6) \mathbf{T}^{(\frac{1}{2})} \bar{D} \mathbf{T}^{(\frac{1}{2})} [D, \bar{D}] \mathbf{T}^{(\frac{1}{2})} \\
& + 108(c+2) \bar{D} \mathbf{T}^{(\frac{1}{2})} \bar{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \\
& \left. \left. - 108(c+2) \mathbf{T} \mathbf{T} \bar{D} \mathbf{T}^{(\frac{1}{2})} - 36(5c+6) \mathbf{T} \bar{D} \mathbf{T} \mathbf{T}^{(\frac{1}{2})} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{3(c+3)}{5(c-3)c} \mathbf{T}^{(\frac{1}{2})} \overline{D} \mathbf{W}^{(2')} + \frac{9(3c-1)}{5(c-3)c} \overline{D} \mathbf{T}^{(\frac{1}{2})} \mathbf{W}^{(2')} \right. \\
& - \left. \frac{2(c+3)}{5(c-3)} \overline{D} \mathbf{W}^{(\frac{5}{2})} \right) + \frac{4}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \overline{D} \mathbf{W}^{(\frac{5}{2}')} - C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(\mathbf{W}^{(3),1} - i \mathbf{W}^{(3),2} \right) \Big] (Z_2) \\
& + \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(72c^2 D \mathbf{T} \partial \overline{D} \mathbf{T}^{(\frac{1}{2})} \right. \right. \\
& - 72c^2 \overline{D} \mathbf{T} \partial D \mathbf{T}^{(\frac{1}{2})} - 18(c^2 - 9c - 18) \mathbf{T} \partial [D, \overline{D}] \mathbf{T}^{(\frac{1}{2})} \\
& - 18(3c^2 + 7c + 6) \partial \mathbf{T} [D, \overline{D}] \mathbf{T}^{(\frac{1}{2})} + 48c(2c+3) \partial D \mathbf{T} \overline{D} \mathbf{T}^{(\frac{1}{2})} \\
& - 48c(2c+3) \partial \overline{D} \mathbf{T} D \mathbf{T}^{(\frac{1}{2})} + 27(5c+6) \partial \overline{D} \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \\
& + 27(5c+6) \partial D \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} \overline{D} \mathbf{T}^{(\frac{1}{2})} + 162(c+2) \partial \mathbf{T}^{(\frac{1}{2})} \overline{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \\
& - 12(2c+3)(c+6) [D, \overline{D}] \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} - 6(2c+3)(c+6) \partial [D, \overline{D}] \mathbf{T} \mathbf{T}^{(\frac{1}{2})} \\
& - 18(7c+6) \mathbf{T} \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} - 144(2c+3) \partial \mathbf{T} \mathbf{T} \mathbf{T}^{(\frac{1}{2})} \\
& - 162(c+2) \partial \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} + (2c^3 - 57c^2 - 243c - 270) \partial^3 \mathbf{T}^{(\frac{1}{2})} \\
& + 36(c+6) \mathbf{T} \overline{D} \mathbf{T} D \mathbf{T}^{(\frac{1}{2})} + 36(c+6) \mathbf{T} D \mathbf{T} \overline{D} \mathbf{T}^{(\frac{1}{2})} \Big) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(- \frac{3}{(c-3)} \mathbf{T} \mathbf{W}^{(\frac{5}{2})} + \frac{3(2c-9)}{10(c-3)c} \mathbf{T}^{(\frac{1}{2})} \partial \mathbf{W}^{(2')} \right. \\
& + \frac{3(7c-9)}{10(c-3)c} \partial \mathbf{T}^{(\frac{1}{2})} \mathbf{W}^{(2')} + \frac{(c+3)}{10(c-3)} [D, \overline{D}] \mathbf{W}^{(\frac{5}{2})} \\
& - \frac{3}{(c-3)} \overline{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{W}^{(2')} - \frac{3}{(c-3)} D \mathbf{T}^{(\frac{1}{2})} \overline{D} \mathbf{W}^{(2')} \Big) + \frac{6}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \partial \mathbf{W}^{(\frac{5}{2}')} \\
& + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(- \frac{1}{2} \overline{D} (\mathbf{W}^{(3),1} + i \mathbf{W}^{(3),2}) + \frac{1}{2} D (\mathbf{W}^{(3),1} - i \mathbf{W}^{(3),2}) - \frac{1}{2} \mathbf{W}^{(\frac{7}{2}),3} \right) \Big] (Z_2) \\
& + \dots \tag{E.7} \{?\}
\end{aligned}$$

Note that the higher spin multiplets $\mathbf{W}^{(3),1}$ and $\mathbf{W}^{(3),2}$ appear together in the above OPE. According to (E.5), we see that $U(1)$ charge in (E.7) is preserved.

Furthermore, the $\mathcal{N} = 2$ OPE between the $\mathcal{N} = 2$ higher spin-2 current and itself can be summarized by

$$\begin{aligned}
\mathbf{W}^{(2)}(Z_1) \mathbf{W}^{(2)}(Z_2) & = \frac{1}{z_{12}^4} 8c + \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^4} 48 \mathbf{T}(Z_2) \\
& + \frac{\theta_{12}}{z_{12}^3} \frac{1}{(2c-3)} \left[-36 \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} - 24(4c-3) D \mathbf{T} \right] (Z_2) \\
& + \frac{\bar{\theta}_{12}}{z_{12}^3} \frac{1}{(2c-3)} \left[24(4c-3) \overline{D} \mathbf{T} - 36 \mathbf{T}^{(\frac{1}{2})} \overline{D} \mathbf{T}^{(\frac{1}{2})} \right] (Z_2) \\
& + \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^3} \left[48 \partial \mathbf{T} - \frac{18}{(2c-3)} \mathbf{T}^{(\frac{1}{2})} [D, \overline{D}] \mathbf{T}^{(\frac{1}{2})} \right] (Z_2)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{z_{12}^2} \left[\frac{1}{(c+1)(2c-3)} \left(24c \overline{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} - 24(4c+3) \mathbf{T} \mathbf{T} \right. \right. \\
& - \left. 12(2c+3) \partial \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} - 16c(2c+3) [D, \overline{D}] \mathbf{T} - 24c \partial \mathbf{T} \right) \\
& + \left. 2C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \mathbf{W}^{(2')} \right] (Z_2) \\
& + \frac{\theta_{12}}{z_{12}^2} \left[\frac{1}{(c+1)(2c-3)} \left(+ 6c [D, \overline{D}] \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} - 24(4c+3) \mathbf{T} D \mathbf{T} \right. \right. \\
& - \left. 6(5c+6) \partial D \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} - 12c \partial \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \right. \\
& - \left. 8(8c^2-9) \partial D \mathbf{T} \right) + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} D \mathbf{W}^{(2')} \left. \right] (Z_2) \\
& + \frac{\bar{\theta}_{12}}{z_{12}^2} \left[\frac{1}{(c+1)(2c-3)} \left(- 6c \overline{D} \mathbf{T}^{(\frac{1}{2})} [D, \overline{D}] \mathbf{T}^{(\frac{1}{2})} - 24(4c+3) \mathbf{T} \overline{D} \mathbf{T} \right. \right. \\
& - \left. 6(5c+6) \partial \overline{D} \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} - 12c \partial \mathbf{T}^{(\frac{1}{2})} \overline{D} \mathbf{T}^{(\frac{1}{2})} \right. \\
& + \left. 8(8c^2-3c-9) \partial \overline{D} \mathbf{T} \right) + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \overline{D} \mathbf{W}^{(2')} \left. \right] (Z_2) \\
& + \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^2} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(- 24(8c^2+15c+18) \mathbf{T} [D, \overline{D}] \mathbf{T} \right. \right. \\
& + \left. 3(c^2-87c-126) \partial [D, \overline{D}] \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} \right. \\
& - \left. 9(5c^2+5c-6) \partial \mathbf{T}^{(\frac{1}{2})} [D, \overline{D}] \mathbf{T}^{(\frac{1}{2})} + 288(2c+3) \mathbf{T} \overline{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \right. \\
& - \left. 18(31c+42) \overline{D} \mathbf{T} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} - 18(31c+42) D \mathbf{T} \mathbf{T}^{(\frac{1}{2})} \overline{D} \mathbf{T}^{(\frac{1}{2})} \right. \\
& - \left. 144c(5c+6) \overline{D} \mathbf{T} D \mathbf{T} + 18(4c+7)(c+6) \partial \overline{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \right. \\
& - \left. 18(4c+7)(c+6) \partial D \mathbf{T}^{(\frac{1}{2})} \overline{D} \mathbf{T}^{(\frac{1}{2})} - 288(2c+3) \mathbf{T} \mathbf{T} \mathbf{T} \right. \\
& - \left. 54(5c+6) \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} - 288(2c+3) \partial \mathbf{T} \mathbf{T} \right. \\
& + \left. 6(8c^3+17c^2-147c-198) \partial^2 \mathbf{T} - 36c(5c+6) \partial [D, \overline{D}] \mathbf{T} \right) \\
& + \left. C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{6(7c-9)}{5(c-3)c} \mathbf{T} \mathbf{W}^{(2')} - \frac{3(2c-9)}{5(c-3)c} \mathbf{T}^{(\frac{1}{2})} \mathbf{W}^{(\frac{5}{2})} \right. \right. \\
& - \left. \frac{3(c-7)}{10(c-3)} [D, \overline{D}] \mathbf{W}^{(2')} \right) + \frac{12}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \mathbf{W}^{(3)} + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \mathbf{W}^{(3),3} \left. \right] (Z_2) \\
& + \frac{1}{z_{12}} \left(\frac{1}{(c+1)(2c-3)} \left[12c \partial \overline{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} + 12c \partial D \mathbf{T}^{(\frac{1}{2})} \overline{D} \mathbf{T}^{(\frac{1}{2})} \right. \right. \\
& - \left. 24(4c+3) \partial \mathbf{T} \mathbf{T} - 6(2c+3) \partial^2 \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} \right. \\
& - \left. 8c(2c+3) \partial [D, \overline{D}] \mathbf{T} \right] + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \partial \mathbf{W}^{(2')} \left. \right) (Z_2) \\
& + \frac{\theta_{12}}{z_{12}} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(72(3c^2+7c+6) [D, \overline{D}] \mathbf{T} D \mathbf{T} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 18(2c^2 + 9c + 6)\partial D\mathbf{T}^{(\frac{1}{2})}[D, \overline{D}]\mathbf{T}^{(\frac{1}{2})} \\
& - 18(c^2 + 15c + 30)\partial[D, \overline{D}]\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} - 24(2c^2 + 45c + 54)\partial D\mathbf{T}\mathbf{T} \\
& - 72(c^2 - 12c - 24)\partial\mathbf{T}D\mathbf{T} - 3(4c^2 + 3c + 18)\partial^2 D\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} \\
& - 18(c^2 - 12)\partial D\mathbf{T}^{(\frac{1}{2})}\partial\mathbf{T}^{(\frac{1}{2})} - 216(c + 2)\overline{D}\mathbf{T}D\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} \\
& - 432(c + 2)D\mathbf{T}\overline{D}\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} + 18(5c + 6)[D, \overline{D}]\mathbf{T}\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} \\
& + 18(5c + 6)\mathbf{T}\partial D\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} - 18(5c + 6)\mathbf{T}[D, \overline{D}]\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} \\
& - 72c\partial\mathbf{T}\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} + 18(5c + 6)D\mathbf{T}\mathbf{T}^{(\frac{1}{2})}[D, \overline{D}]\mathbf{T}^{(\frac{1}{2})} \\
& + 54(3c + 2)D\mathbf{T}\partial\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} - 36(c + 6)\mathbf{T}\partial\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} \\
& + 288(2c + 3)\mathbf{T}\mathbf{T}D\mathbf{T} + 9(c + 6)\partial^2\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} \\
& - 12c(2c^2 - 3c - 6)\partial^2 D\mathbf{T} \Big) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{6(c + 3)}{5(c - 3)c} \mathbf{T}D\mathbf{W}^{(2')} - \frac{18(3c - 1)}{5(c - 3)c} D\mathbf{T}\mathbf{W}^{(2')} \right. \\
& + \frac{3(c + 3)}{5(c - 3)c} \mathbf{T}^{(\frac{1}{2})}D\mathbf{W}^{(\frac{5}{2})} + \frac{9(3c - 1)}{5(c - 3)c} D\mathbf{T}^{(\frac{1}{2})}\mathbf{W}^{(\frac{5}{2})} \\
& \left. + \frac{3(c - 7)}{5(c - 3)} \partial D\mathbf{W}^{(2')} \right) \\
& - \frac{4}{5}C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} D\mathbf{W}^{(3)} + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} (\mathbf{W}^{(\frac{7}{2}),1} + i\mathbf{W}^{(\frac{7}{2}),2}) \Big] (Z_2) \\
& + \frac{\bar{\theta}_{12}}{z_{12}} \left[\frac{1}{(c + 1)(c + 6)(2c - 3)} \left(- 72(3c^2 + 7c + 6)\overline{D}\mathbf{T}[D, \overline{D}]\mathbf{T} \right. \right. \\
& + 18(c^2 + 15c + 30)\partial[D, \overline{D}]\mathbf{T}^{(\frac{1}{2})}\overline{D}\mathbf{T}^{(\frac{1}{2})} \\
& - 18(2c^2 + 9c + 6)\partial\overline{D}\mathbf{T}^{(\frac{1}{2})}[D, \overline{D}]\mathbf{T}^{(\frac{1}{2})} - 72c^2\partial\mathbf{T}\overline{D}\mathbf{T} \\
& - 48(c^2 + 30c + 36)\partial\overline{D}\mathbf{T}\mathbf{T} - 3(4c^2 + 3c + 18)\partial^2\overline{D}\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} \\
& - 18(c^2 - 12)\partial\overline{D}\mathbf{T}^{(\frac{1}{2})}\partial\mathbf{T}^{(\frac{1}{2})} + 18(5c + 6)[D, \overline{D}]\mathbf{T}\mathbf{T}^{(\frac{1}{2})}\overline{D}\mathbf{T}^{(\frac{1}{2})} \\
& + 18(5c + 6)\overline{D}\mathbf{T}\mathbf{T}^{(\frac{1}{2})}[D, \overline{D}]\mathbf{T}^{(\frac{1}{2})} - 18(5c + 6)\mathbf{T}\partial\overline{D}\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} \\
& + 432(c + 2)\overline{D}\mathbf{T}\overline{D}\mathbf{T}^{(\frac{1}{2})}D\mathbf{T}^{(\frac{1}{2})} + 216(c + 2)D\mathbf{T}\overline{D}\mathbf{T}^{(\frac{1}{2})}\overline{D}\mathbf{T}^{(\frac{1}{2})} \\
& + 72c\partial\mathbf{T}\mathbf{T}^{(\frac{1}{2})}\overline{D}\mathbf{T}^{(\frac{1}{2})} + 36(c + 6)\mathbf{T}\partial\mathbf{T}^{(\frac{1}{2})}\overline{D}\mathbf{T}^{(\frac{1}{2})} \\
& - 18(5c + 6)\mathbf{T}\overline{D}\mathbf{T}^{(\frac{1}{2})}[D, \overline{D}]\mathbf{T}^{(\frac{1}{2})} - 54(3c + 2)\overline{D}\mathbf{T}\partial\mathbf{T}^{(\frac{1}{2})}\mathbf{T}^{(\frac{1}{2})} \\
& - 288(2c + 3)\mathbf{T}\mathbf{T}\overline{D}\mathbf{T} + 9(c + 6)\partial^2\mathbf{T}^{(\frac{1}{2})}\overline{D}\mathbf{T}^{(\frac{1}{2})} \\
& \left. + 24(c^3 + 3c^2 - 3c - 18)\partial^2\overline{D}\mathbf{T} \right) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{18(3c - 1)}{5(c - 3)c} \overline{D}\mathbf{T}\mathbf{W}^{(2')} - \frac{6(c + 3)}{5(c - 3)c} \mathbf{T}\overline{D}\mathbf{W}^{(2')} \right. \\
& \left. - \frac{3(c + 3)}{5(c - 3)c} \mathbf{T}^{(\frac{1}{2})}\overline{D}\mathbf{W}^{(\frac{5}{2})} - \frac{9(3c - 1)}{5(c - 3)c} \overline{D}\mathbf{T}^{(\frac{1}{2})}\mathbf{W}^{(\frac{5}{2})} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{3(c-7)}{5(c-3)} \partial \bar{D} \mathbf{W}^{(2')} \Bigg) \\
& + \frac{4}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \bar{D} \mathbf{W}^{(3)} + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} (\mathbf{W}^{(\frac{7}{2}),1} - i \mathbf{W}^{(\frac{7}{2}),2}) \Bigg] (Z_2) \\
& + \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(72(5c+6) \mathbf{T} \partial \bar{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \right. \right. \\
& + 72(5c+6) \mathbf{T} \partial D \mathbf{T}^{(\frac{1}{2})} \bar{D} \mathbf{T}^{(\frac{1}{2})} \\
& + 12(4c+9)(c+6) \partial^2 \bar{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} + 432(c+2) \partial \mathbf{T} \bar{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \\
& + 12(c^2 - 27c - 54) \partial [D, \bar{D}] \mathbf{T}^{(\frac{1}{2})} \partial \mathbf{T}^{(\frac{1}{2})} \\
& + 96c(5c+6) \partial D \mathbf{T} \bar{D} \mathbf{T} + 8(2c^3 + c^2 - 57c - 90) \partial^3 \mathbf{T} \\
& - 12(4c+9)(c+6) \partial^2 D \mathbf{T}^{(\frac{1}{2})} \bar{D} \mathbf{T}^{(\frac{1}{2})} \\
& - 36(5c+6) \mathbf{T} \partial^2 \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} - 72(5c+6) \partial \bar{D} \mathbf{T} \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} \\
& - 432(c+2) \partial \mathbf{T} \partial \mathbf{T} - 96c(5c+6) \partial \bar{D} \mathbf{T} D \mathbf{T} \\
& - 72(5c+6) \bar{D} \mathbf{T} \partial D \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} - 96c^2 \partial [D, \bar{D}] \mathbf{T} \mathbf{T} \\
& - 36(5c+6) \partial \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} - 72(5c+6) D \mathbf{T} \partial \bar{D} \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} \\
& - 432(c+2) \bar{D} \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} D \mathbf{T}^{(\frac{1}{2})} - 72(5c+6) \partial D \mathbf{T} \mathbf{T}^{(\frac{1}{2})} \bar{D} \mathbf{T}^{(\frac{1}{2})} \\
& - 48(4c^2 + 15c + 18) \partial \mathbf{T} [D, \bar{D}] \mathbf{T} - 576(2c+3) \partial \mathbf{T} \mathbf{T} \mathbf{T} \\
& - 36(5c+6) \partial^2 [D, \bar{D}] \mathbf{T}^{(\frac{1}{2})} \mathbf{T}^{(\frac{1}{2})} - 432(c+2) D \mathbf{T} \partial \mathbf{T}^{(\frac{1}{2})} \bar{D} \mathbf{T}^{(\frac{1}{2})} \\
& \left. \left. - 36c(c+2) \partial^2 \mathbf{T}^{(\frac{1}{2})} [D, \bar{D}] \mathbf{T}^{(\frac{1}{2})} \right) \right] \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{18(c-2)}{5(c-3)c} \mathbf{T} \partial \mathbf{W}^{(2')} + \frac{6}{(c-3)} D \mathbf{T} \bar{D} \mathbf{W}^{(2')} \right. \\
& + \frac{3}{(c-3)} D \mathbf{T}^{(\frac{1}{2})} \bar{D} \mathbf{W}^{(\frac{5}{2})} + \frac{6}{(c-3)} \bar{D} \mathbf{T} D \mathbf{W}^{(2')} \\
& + \frac{3}{(c-3)} \bar{D} \mathbf{T}^{(\frac{1}{2})} D \mathbf{W}^{(\frac{5}{2})} + \frac{12(4c-3)}{5(c-3)c} \partial \mathbf{T} \mathbf{W}^{(2')} \\
& - \frac{3(c-12)}{10(c-3)c} \mathbf{T}^{(\frac{1}{2})} \partial \mathbf{W}^{(\frac{5}{2})} - \frac{3(11c-12)}{10(c-3)c} \partial \mathbf{T}^{(\frac{1}{2})} \mathbf{W}^{(\frac{5}{2})} \\
& \left. - \frac{(c-12)}{5(c-3)} \partial [D, \bar{D}] \mathbf{W}^{(2')} \right) + \frac{8}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \partial \mathbf{W}^{(3)} \\
& + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(\frac{1}{2} \partial \mathbf{W}^{(3),3} + \frac{1}{2} \bar{D} (\mathbf{W}^{(\frac{7}{2}),1} + i \mathbf{W}^{(\frac{7}{2}),2}) \right. \\
& \left. \left. - \frac{1}{2} D (\mathbf{W}^{(\frac{7}{2}),1} - i \mathbf{W}^{(\frac{7}{2}),2}) \right) \right] (Z_2) + \dots
\end{aligned} \tag{E.8} \{?\}$$

Note that the higher spin multiplets $\mathbf{W}^{(\frac{7}{2}),1}(Z_2)$ and $\mathbf{W}^{(\frac{7}{2}),2}(Z_2)$ appear together in the above OPE. According to (E.6), we see that $U(1)$ charge in (E.8) is preserved.

F The OPEs between the lowest eight higher spin currents in the component approach corresponding to the section 7

From the $\mathcal{N} = 2$ OPE results in section 6, we obtain its component results.

F.1 The complete 36 OPEs (between the lowest eight higher spin currents) in the component approach

We present the following complete OPEs by reading off the appropriate OPEs from the $\mathcal{N} = 2$ superspace description. The first four types of OPEs are given by

$$\begin{aligned}
\psi^{(\frac{3}{2})}(z) \psi^{(\frac{3}{2})}(w) &= \frac{1}{(z-w)^3} \frac{2c}{3} \\
&+ \frac{1}{(z-w)} \left[\frac{1}{(c+1)(2c-3)} \left(-6cJ^i J^i - 6(c+3)\partial\Psi\Psi + 4c(c+3)T \right) \right. \\
&+ \left. C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \psi^{(2)} \right] (w) + \dots, \\
\psi^{(\frac{3}{2})}(z) \phi^{(2),i}(w) &= \frac{1}{(z-w)^2} \frac{1}{(2c-3)} \left[6cG^i - 18\Psi J^i \right] (w) \\
&+ \frac{1}{(z-w)} \left[\frac{1}{3} \partial(\text{pole two}) + \frac{1}{(c+1)(2c-3)} \left(6ic(\epsilon^{ijk} J^j G^k - \frac{2}{3} i \partial G^i) \right. \right. \\
&- \left. \left. 18(\partial\Psi J^i - \frac{1}{3} \partial(\Psi J^i)) \right) - \frac{1}{2} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \phi^{(\frac{5}{2}),i} \right] (w) + \dots, \\
\psi^{(\frac{3}{2})}(z) \psi^{(\frac{5}{2}),i}(w) &= \frac{1}{(z-w)^3} 6J^i(w) - \frac{1}{(z-w)^2} \frac{18}{(2c-3)} \Psi G^i(w) \\
&+ \frac{1}{(z-w)} \left[\frac{1}{4} \partial(\text{pole two}) \right. \\
&+ \frac{1}{(c+1)(c+6)(2c-3)} \left(18i(5c+6)(\epsilon^{ijk} \Psi J^j G^k - \frac{2}{3} i \Psi \partial G^i) \right. \\
&- 18ic(c+2)(\epsilon^{ijk} G^j G^k - \frac{2}{3} i \partial^2 J^i) + 36(c^2 + 3c + 6)(T J^i - \frac{1}{2} \partial^2 J^i) \\
&- 54(c+2)J^i J^j J^j + 72ic(\epsilon^{ijk} \partial J^j J^k - \frac{1}{3} i \partial^2 J^i) \\
&+ \left. \left. 6(c^2 - 17c - 42)(\partial\Psi G^i - \frac{1}{4} \partial(\Psi G^i)) - 72c\partial\Psi\Psi J^i \right) \right. \\
&+ \left. C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{3(c+3)}{5(c-3)c} \Psi \phi^{(\frac{5}{2}),i} + \frac{9(3c-1)}{5(c-3)c} J^i \psi^{(2)} + \frac{3(c-7)}{10(c-3)} \psi^{(3),i} \right) \right. \\
&+ \left. \left. \frac{2}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \phi^{(3),i} + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \psi^{(3),\alpha=i} \right] (w) + \dots,
\end{aligned}$$

$$\begin{aligned}
\psi^{(\frac{3}{2})}(z) \phi^{(3)}(w) &= \frac{1}{(z-w)^4} 3\Psi(w) - \frac{1}{(z-w)^3} 3\partial\Psi(w) \\
&+ \frac{1}{(z-w)^2} \left[\frac{1}{(c+1)(2c-3)} \left(6(c+3)(T\Psi - \frac{3}{4}\partial^2\Psi) + 36\frac{c(c+1)}{(c+6)} J^i G^i \right. \right. \\
&- \left. \frac{9(13c+18)}{(c+6)} \Psi J^i J^i \right) + \frac{3}{2c} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \Psi \psi^{(2)} + C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \psi^{(\frac{5}{2})} \left. \right] (w) \\
&+ \frac{1}{(z-w)} \left[\frac{1}{5} \partial(\text{pole two}) + \frac{1}{(c+1)(2c-3)} \left(-24(c+3)(\partial\Psi T - \frac{1}{5}\partial(\Psi T)) \right. \right. \\
&- \left. 12c(\partial J^i G^i - \frac{2}{5}\partial(J^i G^i)) + 18(\partial\Psi J^i J^i - \frac{1}{5}\partial(\Psi J^i J^i)) \right) \\
&+ C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{9}{2(c-3)} (\partial\Psi \psi^{(2)} - \frac{1}{5}\partial(\Psi \psi^{(2)})) - \frac{3}{2(c-3)} J^i \phi^{(\frac{5}{2}),i} \right. \\
&- \left. \left. \frac{(c-12)}{5(c-3)} \phi^{(\frac{7}{2})} \right) - \frac{1}{4} C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \phi^{(\frac{7}{2}),i,\alpha=i} \right] (w) + \dots. \tag{F.1} \{?\}
\end{aligned}$$

These OPEs appeared in the section 4.

The next three types of OPEs can be written as

$$\begin{aligned}
\phi^{(2),i}(z) \phi^{(2),j}(w) &= \frac{1}{(z-w)^4} 2\delta^{ij} c + \frac{1}{(z-w)^3} 6i\epsilon^{ijk} J^k(w) \\
&+ \frac{1}{(z-w)^2} \left[\frac{8c(2c+3)}{(c+1)(2c-3)} \delta^{ij} T - \frac{18}{(2c-3)} J^i J^j - \frac{6c}{(c+1)(2c-3)} \delta^{ij} J^k J^k \right. \\
&- \left. \frac{12(2c+3)}{(c+1)(2c-3)} \delta^{ij} \partial\Psi \Psi + \frac{6ic}{(2c-3)} \epsilon^{ijk} \partial J^k + \delta^{ij} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \psi^{(2)} \right] (w) \\
&+ \frac{1}{(z-w)} \left[\frac{4c(2c+3)}{(c+1)(2c-3)} \delta^{ij} \partial T - \frac{6c}{(c+1)(2c-3)} \delta^{ij} \partial J^k J^k \right. \\
&- \frac{6(2c+3)}{(c+1)(2c-3)} \delta^{ij} \partial^2 \Psi \Psi \\
&+ \frac{1}{(c+1)(c+6)(2c-3)} \left(3c(5c+6)(G^i G^j - G^j G^i) - 54i(c+2)\epsilon^{ijk} J^l J^l J^k \right. \\
&- 18(5c+6)\Psi(J^i G^j - J^j G^i) + 18i(5c+6)\epsilon^{ijk} \Psi \partial G^k \\
&+ 12i(4c^2 + 15c + 18)\epsilon^{ijk} T J^k - 18i(5c+6)\epsilon^{ijk} \partial\Psi G^k \\
&- 72ic\epsilon^{ijk} \partial\Psi \Psi J^k - 12(c^2 - 9c - 18)J^i \partial J^j \\
&- \left. 6(c^2 + 39c + 54)\partial J^i J^j + 2i(c^3 - 9c^2 - 45c - 54)\epsilon^{ijk} \partial^2 J^k \right) \\
&+ C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{3i(c+3)}{5(c-3)c} \epsilon^{ijk} \Psi \phi^{(\frac{5}{2}),k} + \frac{9i(3c-1)}{5(c-3)c} \epsilon^{ijk} J^k \psi^{(2)} + \frac{1}{2} \delta^{ij} \partial \psi^{(2)} \right. \\
&- \left. \frac{i(c+3)}{5(c-3)} \epsilon^{ijk} \psi^{(3),k} \right) + \frac{2}{5} i C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \epsilon^{ijk} \phi^{(3),k} + i C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \epsilon^{ijk} \psi^{(3),\alpha=k} \left. \right] (w) + \dots,
\end{aligned}$$

$$\begin{aligned}
\phi^{(2),i}(z) \psi^{(\frac{5}{2}),j} &= \frac{1}{(z-w)^4} 6\delta^{ij} \Psi(w) + \frac{1}{(z-w)^3} \frac{1}{(2c-3)} \left[6i(4c-3)\epsilon^{ijk} G^k - 36i\epsilon^{ijk} \Psi J^k \right] (w) \\
&+ \frac{1}{(z-w)^2} \left[\frac{1}{(c+1)(2c-3)} \left(\frac{72c(c+1)}{(c+6)} \delta^{ij} J^k G^k - 18(c+1) J^i G^j \right. \right. \\
&- \frac{18(13c+18)}{(c+6)} \delta^{ij} \Psi J^k J^k + 24(2c+3) \delta^{ij} T \Psi \\
&- 18(2c+3) \delta^{ij} \partial^2 \Psi - 6c(J^i G^j - J^j G^i) \\
&- 6ic\epsilon^{ijk} \Psi \partial J^k - 12i(2c+3) \epsilon^{ijk} \partial \Psi J^k + 4ic(2c+3) \epsilon^{ijk} \partial G^k \left. \right) \\
&+ \frac{3}{c} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \delta^{ij} \Psi \psi^{(2)} + 2C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \delta^{ij} \psi^{(\frac{5}{2})} - \frac{i}{2} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \epsilon^{ijk} \phi^{(\frac{5}{2}),k} \left. \right] (w) \\
&+ \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-18i(c+6) J^i J^{i+1} G^{i+2} \right. \right. \\
&+ 18i(c+6) J^i J^{i+2} G^{i+1} + 12c(c+6) J^i \partial G^i + 24c^2 \delta^{ij} \sum_k J^k \partial G^k \\
&+ 18(c+6) \Psi \partial J^i J^i - 18(11c+18) \delta^{ij} \sum_k \Psi \partial J^k J^k \\
&- 36(c+1)(c+6) \partial J^i G^i + 36c(c+2) \delta^{ij} \sum_k \partial J^k G^k - 36(c+6) \partial \Psi J^i J^i \\
&- 72c \delta^{ij} \sum_k \partial \Psi J^k J^k - 4(c+6)(2c+3) \delta^{ij} \partial^3 \Psi + 12(2c+3)(c+6) \delta^{ij} \partial T \Psi \left. \right)_{j=i} \\
&+ \frac{1}{(c+1)(c+6)(2c-3)} \left(-72i(2c+3) J^i J^i G^{i+2} + 18i(5c+6) J^i J^{i+2} G^i \right. \\
&- 6(c-18)(2c+3) J^i \partial G^{i+1} - 162i(c+2) J^{i+1} J^{i+1} G^{i+2} \\
&+ 108i(c+2) J^{i+1} J^{i+2} G^{i+1} + 6(4c^2 - 21c - 54) J^{i+1} \partial G^i \\
&- 54i(c+2) J^{i+2} J^{i+2} G^{i+2} + 18(5c+6) \Psi G^i G^{i+1} \\
&+ 18(5c+6) \Psi J^i \partial J^{i+1} - 72c \Psi \partial J^i J^{i+1} + 36i(3c^2 + 7c + 6) T G^{i+2} \\
&- 36i(5c+6) T \Psi J^{i+2} - 6(c^2 + 36c + 36) \partial J^i G^{i+1} - 6c(5c+6) \partial J^{i+1} G^i \\
&- 36(c+6) \partial \Psi J^i J^{i+1} - 54i(3c+2) \partial \Psi \Psi G^{i+2} - 6i(c-18)(2c+3) \partial \Psi \partial J^{i+2} \\
&+ i(2c^3 - 21c^2 - 117c - 162) \partial^2 G^{i+2} - 3i(c+6)(2c+3) \partial^2 \Psi J^{i+2} \left. \right)_{j=i+1} \\
&+ \frac{1}{(c+1)(c+6)(2c-3)} \left(162i(c+2) J^j J^j G^{j+2} - 108i(c+2) J^j J^{j+2} G^j \right. \\
&+ 6(4c^2 - 21c - 54) J^j \partial G^{j+1} \\
&+ 72i(2c+3) J^{j+1} J^{j+1} G^{j+2} - 18i(5c+6) J^{j+1} J^{j+2} G^{j+1} \\
&- 6(c-18)(2c+3) J^{j+1} \partial G^j + 54i(c+2) J^{j+2} J^{j+2} G^{j+2} \\
&- 18(5c+6) \Psi G^j G^{j+1} - 72c \Psi J^j \partial J^{j+1} + 18(5c+6) \Psi \partial J^j J^{j+1} \\
&- 9i(c+6) \Psi \partial^2 J^{j+2} - 36i(3c^2 + 7c + 6) T G^{j+2} + 36i(5c+6) T \Psi J^{j+2} \\
&- 6c(5c+6) \partial J^j G^{j+1} - 6(c^2 + 36c + 36) \partial J^{j+1} G^j - 36(c+6) \partial \Psi J^j J^{j+1}
\end{aligned}$$

$$\begin{aligned}
& + 54i(3c+2)\partial\Psi\Psi G^{j+2} + 6i(2c^2 - 27c - 18)\partial\Psi\partial J^{j+2} + 3i(c+6)(2c+3)\partial^2\Psi J^{j+2} \\
& - i(2c^3 - 21c^2 - 117c - 162)\partial^2 G^{j+2} \Big)_{i=j+1} \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{3}{(c-3)}\delta^{ij}J^k\phi^{(\frac{5}{2}),k} + \frac{6}{(c-3)}J^i\phi^{(\frac{5}{2}),j} + \frac{(c+3)}{5(c-3)}\delta^{ij}\phi^{(\frac{7}{2})} \right. \\
& - \frac{9(3c-1)}{5(c-3)c}(J^i\phi^{(\frac{5}{2}),j} - J^j\phi^{(\frac{5}{2}),i}) - \frac{3i(c+3)}{5(c-3)c}\epsilon^{ijk}\Psi\psi^{(3),k} + \frac{9i(3c-1)}{5(c-3)c}\epsilon^{ijk}G^k\psi^{(2)} \\
& + \frac{9(c-2)}{5(c-3)c}\delta^{ij}\Psi\partial\psi^{(2)} - \frac{6(c+3)}{5(c-3)c}\delta^{ij}\partial\Psi\psi^{(2)} - \frac{i(c+3)}{5(c-3)}\epsilon^{ijk}\partial\psi^{(\frac{5}{2}),k} \Big) \\
& + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(-\frac{1}{2}\delta^{ij}\phi^{(\frac{7}{2}),k,\alpha=k} + \phi^{(\frac{7}{2}),i,\alpha=j} \right) \\
& + C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \left(\frac{4}{5}\delta^{ij}\partial\psi^{(\frac{5}{2})} + \frac{2}{5}i\epsilon^{ijk}\psi^{(\frac{7}{2}),k} \right) \Big] (w) + \dots, \\
\phi^{(2),i}(z)\phi^{(3)}(w) & = \frac{1}{(z-w)^4}12J^i(w) - \frac{1}{(z-w)^3}\frac{18}{(2c-3)}\Psi G^i(w) + \frac{1}{(z-w)^2} \left[\right. \\
& + \frac{1}{(c+1)(c+6)(2c-3)} \left(-9ic(5c+6)\epsilon^{ijk}G^jG^k - 72(2c+3)J^iJ^jJ^j \right. \\
& + 36ic^2\epsilon^{ijk}J^j\partial J^k + 9i(31c+42)\epsilon^{ijk}\Psi J^jG^k \\
& - 3(5c^2 - 57c - 90)\Psi\partial G^i + 12(8c^2 + 15c + 18)TJ^i \\
& + 27(c^2 - 3c - 6)\partial\Psi G^i - 54(5c+6)\partial\Psi\Psi J^i \\
& \left. - 18(3c^2 + 7c + 6)\partial^2 J^i \right) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{3(c-7)}{20(c-3)}\psi^{(3),i} + \frac{3(7c-9)}{5(c-3)c}J^i\psi^{(2)} - \frac{3(2c-9)}{5(c-3)c}\Psi\phi^{(\frac{5}{2}),i} \right) \\
& + \frac{6}{5}C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}\phi^{(3),i} + \frac{1}{2}C_{(\frac{3}{2})(\frac{5}{2})}^{(3)}\psi^{(3),\alpha=i} \Big] (w) \\
& + \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-6ic(5c+6)\epsilon^{ijk}G^j\partial G^k \right. \right. \\
& + 12ic^2\epsilon^{ijk}J^j\partial^2 J^k + 9i(11c+18)\epsilon^{ijk}\Psi J^j\partial G^k \\
& + 9i(11c+18)\epsilon^{ijk}\Psi\partial J^jG^k - 6(c^2 - 9c - 18)\Psi\partial^2 G^i \\
& - 36(5c+6)T\partial J^i + 9i(7c-6)\epsilon^{ijk}\partial\Psi J^jG^k \\
& - 18(5c+6)\partial\Psi\Psi\partial J^i + 18c(c+6)\partial\Psi\partial G^i \\
& + 12(4c^2 + 15c + 18)\partial TJ^i - 18(5c+6)\partial^2\Psi G^i \\
& - 18(5c+6)\partial^2\Psi\Psi J^i + (-17c^2 + 12c + 36)\partial^3 J^i \\
& \left. - 108(c+2)J^i\partial J^jJ^j - 36c\partial J^iJ^jJ^j \right) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{(c+3)}{20(c-3)}\partial\psi^{(3),i} + \frac{3i}{2(c-3)}\epsilon^{ijk}G^j\phi^{(\frac{5}{2}),k} + \frac{3(4c-3)}{5(c-3)c}J^i\partial\psi^{(2)} \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{3i}{2(c-3)} \epsilon^{ijk} J^j \psi^{(3),k} - \frac{9(c-2)}{10(c-3)c} \Psi \partial \phi^{(\frac{5}{2}),i} \\
& - \frac{3(c+3)}{5(c-3)c} \partial J^i \psi^{(2)} + \frac{3(7c+6)}{10(c-3)c} \partial \Psi \phi^{(\frac{5}{2}),i} \Big) + \frac{2}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \partial \phi^{(3),i} \\
& + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(\frac{1}{4} \partial \psi^{(3),\alpha=i} + \frac{i}{4} \epsilon^{ijk} \psi^{(4),j,\alpha=k} \right) \Big] (w) + \dots
\end{aligned} \tag{F.2} \{?\}$$

Note that $(\frac{1}{3}\frac{1}{4}\partial\psi^{(3),\alpha=i} + \frac{i}{4}\epsilon^{ijk}\psi^{(4),j,\alpha=k})(w)$ is a primary current under the stress energy tensor. In the second OPE, we use the notation for the index as follows: $i+2$ becomes 1 for $i=2$, and 2 for $i=3$.

The remaining three types of OPEs are

$$\begin{aligned}
\psi^{(\frac{5}{2}),i}(z) \psi^{(\frac{5}{2}),j}(w) &= \frac{1}{(z-w)^5} 8\delta^{ij} c + \frac{1}{(z-w)^4} 24i\epsilon^{ijk} J^k(w) \\
&+ \frac{1}{(z-w)^3} \left[\frac{1}{(c+1)(2c-3)} \left(4(20c^2+15c-9)\delta^{ij} T - 12(4c+3)\delta^{ij} J^k J^k \right. \right. \\
&- 12(7c+9)\delta^{ij} \partial \Psi \Psi + 36(c+1)J^i J^j + 6i(4c-9)(c+1)\epsilon^{ijk} \partial J^k \Big) \\
&+ \left. 2C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \delta^{ij} \psi^{(2)} \right] (w) \\
&+ \frac{1}{(z-w)^2} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(2c(20c^2+57c+54)\delta^{ij} \partial T \right. \right. \\
&- 12(4c+3)(c+6)\delta^{ij} \partial J^k J^k \\
&- 6(7c+9)(c+6)\delta^{ij} \partial^2 \Psi \Psi + 6(26c^2+9c-18)G^i G^j \\
&- 144i(2c+3)\epsilon^{ijk} J^l J^l J^k - 12(4c^2-63c-90)J^i \partial J^j \\
&- 18(31c+42)\Psi(J^i G^j - J^j G^i) - 6i(4c^2-63c-90)\epsilon^{ijk} \Psi \partial G^k \\
&+ 12i(20c^2+57c+54)\epsilon^{ijk} T J^k + 12(7c^2-42c-72)\partial J^i J^j \\
&+ 18i(4c^2-c-6)\epsilon^{ijk} \partial \Psi G^k - 108i(5c+6)\epsilon^{ijk} \partial \Psi \Psi J^k \\
&+ \left. 2i(4c^3-63c^2-234c-216)\epsilon^{ijk} \partial^2 J^k \right) \\
&+ C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\delta^{ij} \partial \psi^{(2)} - \frac{i(c+3)}{5(c-3)} \epsilon^{ijk} \psi^{(3),k} - \frac{6i(2c-9)}{5(c-3)c} \epsilon^{ijk} \Psi \phi^{(\frac{5}{2}),k} \right. \\
&+ \left. \frac{6i(7c-9)}{5(c-3)c} \epsilon^{ijk} J^k \psi^{(2)} \right) + \frac{12}{5} i C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \epsilon^{ijk} \phi^{(3),k} + i \epsilon^{ijk} C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \psi^{(3),\alpha=k} \Big] (w) \\
&+ \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-324i(c+2)\delta^{ij} J^i G^{i+1} G^{i+2} \right. \right. \\
&- 36i(c+6)J^i J^{i+1} \partial J^{i+2} + 36i(c+6)J^i \partial J^{i+1} J^{i+2} + 144i(2c+3)J^{i+1} G^i G^{i+2} \\
&+ 18(5c+6)\delta^{ij} \sum_k \Psi J^k \partial G^k - 54(3c+2)\delta^{ij} \sum_k \Psi \partial J^k G^k - 144i(2c+3)J^{i+2} G^i G^{i+1} \\
&- \left. 18(c+6)\Psi J^i \partial G^i - 18(c+6)\Psi \partial J^i G^i \right)
\end{aligned}$$

$$\begin{aligned}
& + 72(3c^2 + 7c + 6)\delta^{ij}TT - 36(c + 6)TJ^iJ^i \\
& - 144(2c + 3)\delta^{ij}\sum_k TJ^kJ^k - 72(7c + 6)\delta^{ij}T\partial\Psi\Psi \\
& - 6(8c + 9)(c + 6)\partial G^iG^i + 18(3c^2 - 10c - 24)\delta^{ij}\sum_k \partial G^kG^k \\
& - 18c(c + 6)\partial J^i\partial J^i - 18(c^2 - 7c - 6)\delta^{ij}\sum_k \partial J^k\partial J^k \\
& + 90(c + 6)\partial\Psi J^iG^i + 54(c - 2)\delta^{ij}\sum_k \partial\Psi J^kG^k \\
& + 6(5c + 3)(c + 6)\partial^2 J^iJ^i - 6(c + 6)(2c - 3)\delta^{ij}\sum_k \partial^2 J^kJ^k - 18c(c + 6)\delta^{ij}\partial^2\Psi\partial\Psi \\
& - 6(2c^2 - 13c - 6)\delta^{ij}\partial^3\Psi\Psi + 6(2c + 3)(c^2 - 3c + 18)\delta^{ij}\partial^2T \Big)_{j=i} \\
& + \frac{1}{(c + 1)(c + 6)(2c - 3)} \Big(6(17c^2 + 33c + 18)G^i\partial G^{i+1} \\
& + 18i(c + 6)J^iG^iG^{i+2} - 18i(c + 6)J^{i+1}G^{i+1}G^{i+2} \\
& + 6i(20c^2 + 57c + 54)\partial T J^{i+2} - 36(c + 6)T J^iJ^{i+1} \\
& + 24i(5c^2 + 15c + 18)T\partial J^{i+2} + 18(3c^2 - 8c - 12)\partial G^iG^{i+1} \\
& - 18i(7c + 6)J^iJ^i\partial J^{i+2} - 9(2c^2 - 47c - 66)J^i\partial^2 J^{i+1} \\
& - 18i(17c + 30)\partial J^iJ^iJ^{i+2} - 162i(c + 2)J^{i+1}J^{i+1}\partial J^{i+2} \\
& - 18c(c + 6)\partial J^i\partial J^{i+1} - 54i(5c + 6)\partial J^{i+1}J^{i+1}J^{i+2} \\
& - 216i(2c + 3)\partial J^{i+2}J^{i+2}J^{i+2} \\
& + 3(16c^2 - 69c - 126)\partial^2 J^iJ^{i+1} + i(2c^3 - 77c^2 - 249c - 234)\partial^3 J^{i+2} \\
& - 144(2c + 3)\Psi J^i\partial G^{i+1} + 54(5c + 6)\Psi J^{i+1}\partial G^i \\
& - 144(2c + 3)\Psi\partial J^iG^{i+1} + 54(5c + 6)\Psi\partial J^{i+1}G^i \\
& - 3i(4c^2 - 63c - 90)\Psi\partial^2 G^{i+2} - 18(13c + 6)\partial\Psi J^iG^{i+1} \\
& + 324(c + 2)\partial\Psi J^{i+1}G^i - 54i(5c + 6)\partial\Psi\Psi\partial J^{i+2} \\
& + 12i(2c + 3)(c + 6)\partial\Psi\partial G^{i+2} + 9i(4c^2 - c - 6)\partial^2\Psi G^{i+2} \\
& - 54i(5c + 6)\partial^2\Psi\Psi J^{i+2} \Big)_{j=i+1} \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \Big(-\frac{9(3c - 1)}{5(c - 3)c}\delta^{ij}G^k\phi^{(\frac{5}{2}),k} \\
& - \frac{3(c + 3)}{5(c - 3)c}\delta^{ij}J^k\psi^{(3),k} + \frac{6(c + 3)}{5(c - 3)c}\delta^{ij}\Psi\phi^{(\frac{7}{2})} + \frac{18(3c - 1)}{5(c - 3)c}\delta^{ij}T\psi^{(2)} \\
& + \frac{3(c - 7)}{10(c - 3)}\delta^{ij}\partial^2\psi^{(2)} + \frac{3}{(c - 3)}G^i\phi^{(\frac{5}{2}),j} + \frac{3}{(c - 3)}G^j\phi^{(\frac{5}{2}),i} \\
& + \frac{3}{(c - 3)}J^i\psi^{(3),j} + \frac{3}{(c - 3)}J^j\psi^{(3),i} + \frac{3i(7c - 9)}{5(c - 3)c}\epsilon^{ijk}\partial(J^k\psi^{(2)}) \\
& - \frac{3i(2c - 9)}{5(c - 3)c}\epsilon^{ijk}\partial(\Psi\phi^{(\frac{5}{2}),k}) - \frac{i(c + 3)}{10(c - 3)}\epsilon^{ijk}\partial\psi^{(3),k} \Big)
\end{aligned}$$

$$\begin{aligned}
& + C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \left(\frac{6}{5} \epsilon^{ijk} \partial \phi^{(3),k} + \frac{4}{5} \delta^{ij} \phi^{(4)} \right) \\
& + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(\frac{1}{2} \epsilon^{ijk} \partial \psi^{(3),\alpha=k} + \frac{1}{2} \psi^{(4),i,\alpha=j} + \frac{1}{2} \psi^{(4),j,\alpha=i} \right) \Big] (w) + \dots, \\
\psi^{(\frac{5}{2}),i}(z) \phi^{(3)}(w) & = \frac{1}{(z-w)^4} \frac{1}{(2c-3)} \left[3(20c-21)G^i - 54\Psi J^i \right] (w) \\
& + \frac{1}{(z-w)^3} \left[\frac{1}{(c+1)(2c-3)} \left(6i(4c+3)\epsilon^{ijk} J^j G^k - 6c\Psi \partial J^i \right. \right. \\
& - \left. 6(7c+9)\partial\Psi J^i + (20c^2+15c-9)\partial G^i \right) - \frac{1}{2} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \phi^{(\frac{5}{2}),i} \Big] (w) \\
& + \frac{1}{(z-w)^2} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(9(37c+54)J^i J^j G^j - 252(2c+3)J^j J^j G^i \right. \right. \\
& + 6i(11c^2-84c-144)\epsilon^{ijk} J^j \partial G^k - 54i(3c+4)\epsilon^{ijk}\Psi G^j G^k \\
& - \frac{63}{2}i(5c+6)\epsilon^{ijk}\Psi J^j \partial J^k + \frac{63}{2}i(5c+6)\epsilon^{ijk}\Psi \partial J^j J^k \\
& + \frac{3}{2}(8c+15)(c+6)\Psi \partial^2 J^i + 6(55c^2+87c+54)TG^i \\
& - 216(3c+4)T\Psi J^i - 3i(25c^2-138c-216)\epsilon^{ijk}\partial J^j G^k \\
& - 126(5c+6)\partial\Psi\Psi G^i - 6(14c^2-57c-90)\partial\Psi \partial J^i \\
& + \left. 3(7c+12)(c+6)\partial^2\Psi J^i + \frac{1}{2}(10c^3-147c^2-603c-702)\partial^2 G^i \right) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{3(23c-21)}{10(c-3)c} G^i \psi^{(2)} + \frac{3i(13c-21)}{10(c-3)c} \epsilon^{ijk} J^j \phi^{(\frac{5}{2}),k} \right. \\
& + \left. \frac{9(c-7)}{10(c-3)c} \Psi \psi^{(3),i} - \frac{(c+3)}{5(c-3)} \partial \phi^{(\frac{5}{2}),i} \right) \\
& + \left. \frac{7}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \psi^{(\frac{7}{2}),i} - \frac{1}{2} i C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \epsilon^{ijk} \phi^{(\frac{7}{2}),j,\alpha=k} \right] (w) \\
& + \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(9(17c+30)J^i J^j \partial G^j - 108(2c+3)J^j J^j \partial G^i \right. \right. \\
& + 9(17c+30)\partial J^i J^j G^j - 216(2c+3)\partial J^j J^j G^i \\
& + 9(13c+6)J^i \partial J^j G^j + 3i(2c+3)(5c-42)\epsilon^{ijk} J^j \partial^2 G^k \\
& - 36i(2c+3)\epsilon^{ijk}\Psi \partial(G^j G^k) - 144(2c+3)T\Psi \partial J^i \\
& - 216cT\partial\Psi J^i + 6(19c^2+3c-18)T\partial G^i \\
& - 3i(7c+6)(c+6)\epsilon^{ijk}\partial J^j \partial G^k - 54ic\epsilon^{ijk}\partial\Psi G^j G^k \\
& - 27i(5c+6)\epsilon^{ijk}\partial\Psi J^j \partial J^k - 54(5c+6)\partial\Psi\Psi \partial G^i \\
& + 54(3c^2+7c+6)\partial TG^i - 144(2c+3)\partial T\Psi J^i \\
& - 9i(3c^2-23c-30)\epsilon^{ijk}\partial^2 J^j G^k - 54(5c+6)\partial^2\Psi\Psi G^i \\
& + \left. \frac{1}{2}(2c^3-67c^2-183c-198)\partial^3 G^i - 27i(5c+6)\epsilon^{ijk}\Psi J^j \partial^2 J^k \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{3}{2}(4c^2 + 3c + 18)\Psi\partial^3 J^i - 9(4c^2 - 7c - 6)\partial\Psi\partial^2 J^i \\
& - 9(c^2 - 39c - 54)\partial^2\Psi\partial J^i + 3(3c - 2)(c + 6)\partial^3\Psi J^i \Big) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{9(4c - 3)}{10(c - 3)c} G^i \partial\psi^{(2)} - \frac{3i}{2(c - 3)} \epsilon^{ijk} G^j \psi^{(3),k} \right. \\
& - \frac{3}{(c - 3)} J^i \phi^{(\frac{7}{2})} + \frac{3(2c - 9)}{10(c - 3)c} \Psi \partial\psi^{(3),i} - \frac{3}{(c - 3)} T \phi^{(\frac{5}{2}),i} \\
& + \frac{3i(2c - 9)}{10(c - 3)c} \epsilon^{ijk} \partial J^j \phi^{(\frac{5}{2}),k} + \frac{3(7c - 9)}{10(c - 3)c} \partial G^i \psi^{(2)} \\
& - \frac{9(c + 3)}{10(c - 3)c} \partial\Psi \psi^{(3),i} - \frac{(c + 3)}{20(c - 3)} \partial^2 \phi^{(\frac{5}{2}),i} + \frac{3i(7c - 9)}{10(c - 3)c} \epsilon^{ijk} J^j \partial\phi^{(\frac{5}{2}),k} \Big) \\
& + \frac{3}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \partial\psi^{(\frac{7}{2}),i} + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(-\frac{1}{4} i \epsilon^{ijk} \partial\phi^{(\frac{7}{2}),j,\alpha=k} - \frac{1}{2} \phi^{(\frac{9}{2}),\alpha=i} \right) \Big] (w) + \dots, \\
\phi^{(3)}(z) \phi^{(3)}(w) & = \frac{1}{(z - w)^6} 10c + \frac{1}{(z - w)^4} \left[\frac{1}{(c + 1)(2c - 3)} \left(6(20c^2 + 7c - 15)T \right. \right. \\
& - 9(4c + 3)J^i J^i - 18(5c + 6)\partial\Psi\Psi \Big) + \frac{3}{2} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \psi^{(2)} \Big] (w) \\
& + \frac{1}{(z - w)^3} \left[\frac{1}{2} \partial(\text{pole four}) \right] (w) \\
& + \frac{1}{(z - w)^2} \left[\frac{1}{(c + 1)(c + 6)(2c - 3)} \left(-144i(2c + 3)\epsilon^{ijk} J^i G^j G^k \right. \right. \\
& - 216(5c + 6)T\partial\Psi\Psi + 36(5c + 6)\Psi J^i \partial G^i - 72(5c + 6)\Psi \partial J^i G^i \\
& + 12(7c^2 - 42c - 72)\partial G^i G^i - \frac{9}{2}(12c^2 - 35c - 66)\partial J^i \partial J^i \\
& + 36(5c + 6)\partial\Psi J^i G^i + \frac{9}{2}(4c + 11)(c + 6)\partial^2 J^i J^i + 24(16c^2 + 27c + 18)TT \\
& - \frac{27}{2}(5c + 6)(c + 6)\partial^2\Psi\partial\Psi - \frac{3}{2}(c - 42)(5c + 6)\partial^3\Psi\Psi \\
& + \frac{9}{2}(4c^3 - 17c^2 + 63c + 126)\partial^2 T - 288(2c + 3)T J^i J^i \Big) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{3(11c - 12)}{10(c - 3)c} G^i \phi^{(\frac{5}{2}),i} + \frac{9(c - 2)}{5(c - 3)c} J^i \psi^{(3),i} + \frac{9(c - 7)}{40(c - 3)} \partial^2 \psi^{(2)} \right. \\
& - \frac{3(c - 12)}{5(c - 3)c} \Psi \phi^{(\frac{7}{2})} + \frac{12(4c - 3)}{5(c - 3)c} T \psi^{(2)} \Big) + \frac{8}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \phi^{(4)} + \frac{1}{4} C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \psi^{(4),i,\alpha=i} \Big] (w) \\
& + \frac{1}{(z - w)} \left[-\frac{1}{24} \partial^3(\text{pole four}) + \frac{1}{2} \partial(\text{pole two}) \right] (w) + \dots. \tag{F.3} \{?\}
\end{aligned}$$

In the first OPE, we use the same convention for the index as before. One sees that $(-\frac{1}{7}\frac{1}{4}i\epsilon^{ijk}\partial\phi^{(\frac{7}{2}),j,\alpha=k} - \frac{1}{2}\phi^{(\frac{9}{2}),\alpha=i})(w)$ is a primary current under the stress energy tensor. We

can also describe the above OPEs in the manifest way of $U(1)$ charge. Any three higher spin currents having $SO(3)$ index i can be decomposed into the one with $U(1)$ charge $+1$, the one with $U(1)$ charge -1 , and the one with vanishing $U(1)$ charge. In the large c limit, all the nonlinear terms on the right hand side in (F.3) disappear.

F.2 The 8 OPEs between the eight higher spin currents and the lowest higher spin- $\frac{3}{2}$ current

As emphasized in section 7, it is very useful to write down the following eight OPEs (including the first OPE of (F.1)), by taking the OPEs between the eight higher spin currents located at z coordinate and the lowest higher spin current located at w coordinate,

$$\begin{aligned}
\phi^{(2),i}(z) \psi^{(\frac{3}{2})}(w) &= \frac{1}{(z-w)^2} \frac{1}{(2c-3)} \left[6cG^i - 18\Psi J^i \right] (w) \\
&+ \frac{1}{(z-w)} \left[\frac{2}{3} \partial(\text{pole two}) - \frac{1}{(c+1)(2c-3)} \left(6ic(\epsilon^{ijk} J^j G^k - \frac{2}{3} i \partial G^i) \right. \right. \\
&- \left. \left. 18(\partial \Psi J^i - \frac{1}{3} \partial(\Psi J^i)) \right) + \frac{1}{2} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \phi^{(\frac{5}{2}),i} \right] (w) + \dots, \\
\psi^{(\frac{5}{2}),i}(z) \psi^{(\frac{3}{2})}(w) &= \frac{1}{(z-w)^3} 6J^i(w) + \frac{1}{(z-w)^2} \left[\frac{18}{(2c-3)} \Psi G^i + 6\partial J^i \right] (w) \\
&+ \frac{1}{(z-w)} \left[\frac{3}{4} \partial(\text{pole two}) \right. \\
&+ \frac{1}{(c+1)(c+6)(2c-3)} \left(18i(5c+6)(\epsilon^{ijk} \Psi J^j G^k - \frac{2}{3} i \Psi \partial G^i) \right. \\
&- \left. 18ic(c+2)(\epsilon^{ijk} G^j G^k - \frac{2}{3} i \partial^2 J^i) + 36(c^2+3c+6)(TJ^i - \frac{1}{2} \partial^2 J^i) \right. \\
&- \left. 54(c+2)J^i J^j J^j + 72ic(\epsilon^{ijk} \partial J^j J^k - \frac{1}{3} i \partial^2 J^i) \right. \\
&+ \left. 6(c^2-17c-42)(\partial \Psi G^i - \frac{1}{4} \partial(\Psi G^i)) - 72c \partial \Psi \Psi J^i \right] - \frac{3}{2} \partial^2 J^i \\
&+ C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{3(c+3)}{5(c-3)c} \Psi \phi^{(\frac{5}{2}),i} + \frac{9(3c-1)}{5(c-3)c} J^i \psi^{(2)} + \frac{3(c-7)}{10(c-3)} \psi^{(3),i} \right) \\
&+ \left. \frac{2}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \phi^{(3),i} + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \psi^{(3),\alpha=i} \right] (w) + \dots, \\
\phi^{(3)}(z) \psi^{(\frac{3}{2})}(w) &= \frac{1}{(z-w)^4} 3\Psi(w) + \frac{1}{(z-w)^3} 6\partial \Psi(w) \\
&+ \frac{1}{(z-w)^2} \left[\frac{1}{(c+1)(2c-3)} \left(6(c+3)(T\Psi - \frac{3}{4} \partial^2 \Psi) + 36 \frac{c(c+1)}{(c+6)} J^i G^i \right. \right. \\
&- \left. \left. \frac{9(13c+18)}{(c+6)} \Psi J^i J^i \right) + \frac{9}{2} \partial^2 \Psi + \frac{3}{2c} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \Psi \psi^{(2)} + C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \psi^{(\frac{5}{2})} \right] (w)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{(z-w)} \left[\frac{4}{5} \partial(\text{pole two}) + \frac{1}{(c+1)(2c-3)} \left(24(c+3)(\partial\Psi T - \frac{1}{5}\partial(\Psi T)) \right. \right. \\
& + 12c(\partial J^i G^i - \frac{2}{5}\partial(J^i G^i)) - 18(\partial\Psi J^i J^i - \frac{1}{5}\partial(\Psi J^i J^i)) \left. \left. \right) - \frac{8}{5}\partial^3\Psi \right. \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{9}{2(c-3)}(\partial\Psi\psi^{(2)} - \frac{1}{5}\partial(\Psi\psi^{(2)})) + \frac{3}{2(c-3)}J^i\phi^{(\frac{5}{2}),i} \right. \\
& + \left. \left. \frac{(c-12)}{5(c-3)}\phi^{(\frac{7}{2})} \right) + \frac{1}{4}C_{(\frac{3}{2})(\frac{5}{2})}^{(3)}\phi^{(\frac{7}{2}),i,\alpha=i} \right] (w) + \dots. \tag{F.4} \{?\}
\end{aligned}$$

That is, the OPE (F.1) and the above three OPEs (F.4) will appear in the $\mathcal{N} = 3$ version (7.2) in appropriate supersymmetric way as in [59].

G The OPEs between the lowest eight higher spin currents in the component approach corresponding to the section 8

Because the higher spin currents after decoupling the spin- $\frac{1}{2}$ current of $\mathcal{N} = 3$ superconformal algebra are determined via (8.4), we can calculate the OPEs between them.

G.1 The complete 36 OPEs (between the lowest eight higher spin currents) in the component approach

Based on the results of Appendix F, the OPEs between the composite fields appearing on the right hand sides of (8.4) are known, we calculate the OPEs between the higher spin currents after factoring out the spin- $\frac{1}{2}$ current and they are described as follows. The first four types of OPEs (totally eight OPEs) can be summarized as

$$\begin{aligned}
\hat{\psi}^{(\frac{3}{2})}(z) \hat{\psi}^{(\frac{3}{2})}(w) &= \frac{1}{(z-w)^3} \frac{2c}{3} \\
&+ \frac{1}{(z-w)} \left[\frac{1}{(c+1)(2c-3)} \left(-6c\hat{J}^i\hat{J}^i + 4c(c+3)\hat{T} \right) + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\hat{\psi}^{(2)} \right] (w) + \dots, \\
\hat{\psi}^{(\frac{3}{2})}(z) \hat{\phi}^{(2),i}(w) &= \frac{1}{(z-w)^2} \frac{6c}{(2c-3)} \hat{G}^i(w) \\
&+ \frac{1}{(z-w)} \left[\frac{1}{(c+1)(2c-3)} \left(2c(c+3)\partial\hat{G}^i + 6ic\epsilon^{ijk}\hat{J}^j\hat{G}^k \right) - \frac{1}{2}C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\hat{\phi}^{(\frac{5}{2}),i} \right] (w) \\
&+ \dots, \\
\hat{\psi}^{(\frac{3}{2})}(z) \hat{\psi}^{(\frac{5}{2}),i}(w) &= \frac{1}{(z-w)^3} 6\hat{J}^i(w) \\
&+ \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-18ic(c+2)\epsilon^{ijk}\hat{G}^j\hat{G}^k + 36(c^2+3c+6)\hat{T}\hat{J}^i \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - 6(5c^2 + 9c + 18)\partial^2 \hat{J}^i - 54(c+2)\hat{J}^i \hat{J}^j \hat{J}^j + 18i(c-6)\epsilon^{ijk}\partial \hat{J}^j \hat{J}^k \Big) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{9(3c-1)}{5(c-3)c} \hat{J}^i \hat{\psi}^{(2)} + \frac{3(c-7)}{10(c-3)} \hat{\psi}^{(3),i} \right) + \frac{2}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \hat{\phi}^{(3),i} \\
& + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \hat{\psi}^{(3),\alpha=i} \Big] (w) + \dots, \\
\hat{\psi}^{(\frac{3}{2})}(z) \hat{\phi}^{(3)}(w) &= \frac{1}{(z-w)^2} \left[\frac{36c}{(c+1)(2c-3)} \hat{J}^i \hat{G}^i + C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \hat{\psi}^{(\frac{5}{2})} \right] (w) \\
& + \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-36c\partial \hat{J}^i \hat{G}^i + 12c(c+3)\hat{J}^i \partial \hat{G}^i \right) \right. \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{3}{2(c-3)} \hat{J}^i \hat{\phi}^{(\frac{5}{2}),i} - \frac{(c-12)}{5(c-3)} \hat{\phi}^{(\frac{7}{2})} \right) + \frac{1}{5} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \partial \hat{\psi}^{(\frac{5}{2})} \\
& \left. - \frac{1}{4} C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \hat{\phi}^{(\frac{7}{2}),i,\alpha=i} \right] (w) + \dots. \tag{G.1} \{?\}
\end{aligned}$$

In particular, the $\Psi(w)$ dependent-terms with $C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}$ structure constant appearing in Appendix *F* do not appear in the third and fourth equations of (G.1).

The next three types of OPEs can be described as

$$\begin{aligned}
\hat{\phi}^{(2),i}(z) \hat{\phi}^{(2),j}(w) &= \frac{1}{(z-w)^4} 2c \delta^{ij} + \frac{1}{(z-w)^3} 6i\epsilon^{ijk} \hat{J}^k (w) \\
& + \frac{1}{(z-w)^2} \left[\frac{8c(2c+3)}{(c+1)(2c-3)} \delta^{ij} \hat{T} - \frac{18}{(2c-3)} \hat{J}^i \hat{J}^j - \frac{6c}{(c+1)(2c-3)} \delta^{ij} \hat{J}^k \hat{J}^k \right. \\
& + \frac{6ic}{(2c-3)} \epsilon^{ijk} \partial \hat{J}^k + \delta^{ij} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \hat{\psi}^{(2)} \Big] (w) \\
& + \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(6c(5c+6) \hat{G}^i \hat{G}^j - 54i(c+2) \epsilon^{ijk} \hat{J}^l \hat{J}^l \hat{J}^k \right) \right. \\
& + 2c(4c^2 + 15c + 18) \delta^{ij} \partial \hat{T} - 6c(c+6) \delta^{ij} \partial \hat{J}^k \hat{J}^k \\
& + 12i(4c^2 + 15c + 18) \epsilon^{ijk} \hat{T} \hat{J}^k - 12(c^2 - 9c - 18) \hat{J}^i \partial \hat{J}^j \\
& \left. - 6(c^2 + 24c + 36) \partial \hat{J}^i \hat{J}^j + 2i(c^3 - 9c^2 - 45c - 54) \epsilon^{ijk} \partial^2 \hat{J}^k \right) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{9i(3c-1)}{5(c-3)c} \epsilon^{ijk} \hat{J}^k \hat{\psi}^{(2)} + \frac{1}{2} \delta^{ij} \partial \hat{\psi}^{(2)} - \frac{i(c+3)}{5(c-3)} \epsilon^{ijk} \hat{\psi}^{(3),k} \right) \\
& + \frac{2}{5} i C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \epsilon^{ijk} \hat{\phi}^{(3),k} + i C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \epsilon^{ijk} \hat{\psi}^{(3),\alpha=k} \Big] (w) + \dots, \\
\hat{\phi}^{(2),i}(z) \hat{\psi}^{(\frac{5}{2}),j}(w) &= \frac{1}{(z-w)^3} \frac{1}{(2c-3)} \left[6i(4c-3) \epsilon^{ijk} \hat{G}^k \right] (w) \\
& + \frac{1}{(z-w)^2} \left[\frac{1}{(c+1)(2c-3)} \left(\frac{72c(c+1)}{(c+6)} \delta^{ij} \hat{J}^k \hat{G}^k - 18(c+1) \hat{J}^i \hat{G}^j \right) \right.
\end{aligned}$$

$$\begin{aligned}
& - 6c(\hat{J}^i \hat{G}^j - \hat{J}^j \hat{G}^i) + 4ic(2c+3)\epsilon^{ijk}\partial\hat{G}^k \Big) \\
& + 2C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}\delta^{ij}\hat{\psi}^{(\frac{5}{2})} - \frac{i}{2}C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\epsilon^{ijk}\hat{\phi}^{(\frac{5}{2}),k} \Big] (w) \\
& + \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-18i(c+6)\hat{J}^i(\hat{J}^{i+1}\hat{G}^{i+2} - \hat{J}^{i+2}\hat{G}^{i+1}) \right. \right. \\
& + 24c^2\delta^{ij}\sum_k \hat{J}^k\partial\hat{G}^k + 36c(c+2)\delta^{ij}\sum_k \partial\hat{J}^k\hat{G}^k + 12c(c+6)\hat{J}^i\partial\hat{G}^i \\
& - 36(c+1)(c+6)\partial\hat{J}^i\hat{G}^i \Big)_{j=i} \\
& + \frac{1}{(c+1)(c+6)(2c-3)} \left(-162i(c+2)\hat{J}^{i+1}\hat{J}^{i+1}\hat{G}^{i+2} \right. \\
& + 108i(c+2)\hat{J}^{i+1}\hat{J}^{i+2}\hat{G}^{i+1} + 36i(3c^2+7c+6)\hat{T}\hat{G}^{i+2} \\
& - 6(c^2+36c+36)\partial\hat{J}^i\hat{G}^{i+1} - 6(c-18)(2c+3)\hat{J}^i\partial\hat{G}^{i+1} \\
& - 6c(5c+6)\partial\hat{J}^{i+1}\hat{G}^i + 6(4c^2-21c-54)\hat{J}^{i+1}\partial\hat{G}^i \\
& - 72i(2c+3)\hat{J}^i\hat{J}^i\hat{G}^{i+2} + 18i(5c+6)\hat{J}^i\hat{J}^{i+2}\hat{G}^i - 54i(c+2)\hat{J}^{i+2}\hat{J}^{i+2}\hat{G}^{i+2} \\
& + i(2c^3-21c^2-117c-162)\partial^2\hat{G}^{i+2} \Big)_{j=i+1} \\
& + \frac{1}{(c+1)(c+6)(2c-3)} \left(162i(c+2)\hat{J}^j\hat{J}^j\hat{G}^{j+2} - 108i(c+2)\hat{J}^j\hat{J}^{j+2}\hat{G}^j \right. \\
& + 6(4c^2-21c-54)\hat{J}^j\partial\hat{G}^{j+1} + 72i(2c+3)\hat{J}^{j+1}\hat{J}^{j+1}\hat{G}^{j+2} \\
& - 18i(5c+6)\hat{J}^{j+1}\hat{J}^{j+2}\hat{G}^{j+1} - 6(c-18)(2c+3)\hat{J}^{j+1}\partial\hat{G}^j \\
& + 54i(c+2)\hat{J}^{j+2}\hat{J}^{j+2}\hat{G}^{j+2} - 36i(3c^2+7c+6)\hat{T}\hat{G}^{j+2} \\
& - 6c(5c+6)\partial\hat{J}^j\hat{G}^{j+1} - 6(c^2+36c+36)\partial\hat{J}^{j+1}\hat{G}^j \\
& - i(2c^3-21c^2-117c-162)\partial^2\hat{G}^{j+2} \Big)_{j=i-1} \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{3}{(c-3)}\delta^{ij}\hat{J}^k\hat{\phi}^{(\frac{5}{2}),k} + \frac{6}{(c-3)}\hat{J}^i\hat{\phi}^{(\frac{5}{2}),j} + \frac{(c+3)}{5(c-3)}\delta^{ij}\hat{\phi}^{(\frac{7}{2})} \right. \\
& - \frac{9(3c-1)}{5(c-3)c}(\hat{J}^i\hat{\phi}^{(\frac{5}{2}),j} - \hat{J}^j\hat{\phi}^{(\frac{5}{2}),i}) + \frac{9i(3c-1)}{5(c-3)c}\epsilon^{ijk}\hat{G}^k\hat{\psi}^{(2)} - \frac{i(c+3)}{5(c-3)}\epsilon^{ijk}\partial\hat{\phi}^{(\frac{5}{2}),k} \Big) \\
& + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(-\frac{1}{2}\delta^{ij}\hat{\phi}^{(\frac{7}{2}),k,\alpha=k} + \hat{\phi}^{(\frac{7}{2}),i,\alpha=j} \right) \\
& + C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \left(\frac{4}{5}\delta^{ij}\partial\hat{\psi}^{(\frac{5}{2})} + \frac{2}{5}i\epsilon^{ijk}\hat{\psi}^{(\frac{7}{2}),k} \right) \Big] (w) + \dots, \\
\hat{\phi}^{(2),i}(z)\hat{\phi}^{(3)}(w) & = \frac{1}{(z-w)^4}12\hat{J}^i(w) + \frac{1}{(z-w)^2} \left[
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{(c+1)(c+6)(2c-3)} \left(-9ic(5c+6)\epsilon^{ijk}\hat{G}^j\hat{G}^k - 72(2c+3)\hat{J}^i\hat{J}^j\hat{J}^j \right. \\
& + 9i(4c^2+15c+18)\epsilon^{ijk}\hat{J}^j\partial\hat{J}^k + 12(8c^2+15c+18)\hat{T}\hat{J}^i - 9(6c^2-c-6)\partial^2\hat{J}^i \Big) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{3(c-7)}{20(c-3)}\hat{\psi}^{(3),i} + \frac{3(7c-9)}{5(c-3)c}\hat{J}^i\hat{\psi}^{(2)} \right) \\
& + \frac{6}{5}C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}\hat{\phi}^{(3),i} + \frac{1}{2}C_{(\frac{3}{2})(\frac{5}{2})}^{(3)}\hat{\psi}^{(3),\alpha=i} \Big] (w) \\
& + \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-3ic(5c+6)\epsilon^{ijk}\partial(\hat{G}^j\hat{G}^k) \right. \right. \\
& + 3i(4c^2+15c+18)\epsilon^{ijk}\partial(\hat{J}^j\partial\hat{J}^k) - 108(c+2)\hat{J}^i\partial\hat{J}^j\hat{J}^j - 36c\partial\hat{J}^i\hat{J}^j\hat{J}^j \\
& - 36(5c+6)\hat{T}\partial\hat{J}^i + 12(4c^2+15c+18)\partial\hat{T}\hat{J}^i - (2c-15)(5c+6)\partial^3\hat{J}^i \Big) \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{(c+3)}{20(c-3)}\partial\hat{\psi}^{(3),i} + \frac{3i}{2(c-3)}\epsilon^{ijk}\hat{G}^j\hat{\phi}^{(\frac{5}{2}),k} + \frac{3(4c-3)}{5(c-3)c}\hat{J}^i\partial\hat{\psi}^{(2)} \right. \\
& - \frac{3i}{2(c-3)}\epsilon^{ijk}\hat{J}^j\hat{\psi}^{(3),k} - \frac{3(c+3)}{5(c-3)c}\partial\hat{J}^i\hat{\psi}^{(2)} \Big) + \frac{2}{5}C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}\partial\hat{\phi}^{(3),i} \\
& + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(\frac{1}{4}\partial\hat{\psi}^{(3),\alpha=i} + \frac{i}{4}\epsilon^{ijk}\hat{\psi}^{(4),j,\alpha=k} \right) \Big] (w) + \dots \tag{G.2} \{?\}
\end{aligned}$$

In this case, the $\Psi(w)$ dependent terms appearing in Appendix *F* are disappeared in the corresponding OPEs in (G.2). The presentation in the first-order pole of the OPE between the higher spin-2 currents and the higher spin- $\frac{5}{2}$ currents is rather complicated. For the indices $(i, j) = (1, 1), (2, 2)$ and $(3, 3)$ of these OPEs, we can cover them from the first piece with $j = i$ condition of the first-order pole. In other words, the $(i, j) = (2, 2)$ case can be obtained from the expression of $(i, j) = (1, 1)$ by replacing $1 \rightarrow 2, 2 \rightarrow 3$, and $3 \rightarrow 1$ simply. The $(i, j) = (3, 3)$ case can be obtained similarly from the result of $(i, j) = (1, 1)$ by changing with $1 \rightarrow 3, 2 \rightarrow 1$, and $3 \rightarrow 2$.

Furthermore, for the indices $(i, j) = (1, 2), (2, 3)$ and $(3, 1)$ of these OPEs can be analyzed with the second piece with $j = i + 1$ condition of the first-order pole. The $(i, j) = (2, 3)$ case can be obtained from the expression of $(i, j) = (1, 2)$ by the above first replacement while the $(i, j) = (3, 1)$ case can be obtained from the expression of $(i, j) = (1, 2)$ by the above second replacement.

For the remaining three cases where the indices are given by $(i, j) = (2, 1), (3, 2)$ and $(1, 3)$, we have the first case $(i, j) = (2, 1)$ from the third piece with $j = i - 1$ condition of the first-order pole. The case $(i, j) = (3, 2)$ can be obtained from this by replacing the indices according to the above first replacement. The final case $(i, j) = (1, 3)$ can be obtained from

the case $(i, j) = (2, 1)$ by the above second replacement.

Now the remaining three types of OPEs can be written as

$$\begin{aligned}
\hat{\psi}^{(\frac{5}{2}),i}(z) \hat{\psi}^{(\frac{5}{2}),j}(w) &= \frac{1}{(z-w)^5} 2(4c-3)\delta^{ij} + \frac{1}{(z-w)^4} \frac{6i(4c-3)}{c} \epsilon^{ijk} \hat{J}^k(w) \\
&+ \frac{1}{(z-w)^3} \left[\frac{1}{(c+1)(2c-3)} \left(4(20c^2+3c-27)\delta^{ij}\hat{T} - 6(8c+3)\delta^{ij}\hat{J}^k\hat{J}^k \right. \right. \\
&+ \left. \frac{18}{c}(c+1)(2c+3)\hat{J}^i\hat{J}^j + 24i(c-3)(c+1)\epsilon^{ijk}\partial\hat{J}^k \right) + \frac{(2c-3)}{c} C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \delta^{ij} \hat{\psi}^{(2)} \Big] (w) \\
&+ \frac{1}{(z-w)^2} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(2(20c^2+3c-27)(c+6)\delta^{ij}\partial\hat{T} \right. \right. \\
&- 6(8c+3)(c+6)\delta^{ij}\partial\hat{J}^k\hat{J}^k \\
&+ \frac{18}{c}(c+1)(c+6)(2c+3)\partial\hat{J}^i\hat{J}^j + 6(13c^2-3c-18)(\hat{G}^i\hat{G}^j - \hat{G}^j\hat{G}^i) \\
&- \frac{3}{c}i(8c^3+3c^2-45c-54)\epsilon^{ijk}\hat{J}^l\hat{J}^l\hat{J}^k + \frac{3}{c}i(8c^3-93c^2-135c+54)\epsilon^{ijk}\hat{J}^k\hat{J}^l\hat{J}^l \\
&+ \frac{12}{c}i(20c^3+45c^2+9c-54)\epsilon^{ijk}\hat{T}\hat{J}^k \\
&+ \left. \frac{i}{c}(8c^4-132c^3-297c^2-189c+162)\epsilon^{ijk}\partial^2\hat{J}^k \right) \\
&+ C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{(2c-3)}{2c} \delta^{ij} \partial\hat{\psi}^{(2)} - \frac{i(c+3)}{5c} \epsilon^{ijk} \hat{\psi}^{(3),k} + \frac{3i(14c-3)}{5c^2} \epsilon^{ijk} \hat{J}^k \hat{\psi}^{(2)} \right) \\
&+ \frac{6i(2c-1)}{5c} C_{(\frac{3}{2})(3)}^{(\frac{5}{2})} \epsilon^{ijk} \hat{\phi}^{(3),k} + \frac{i(c-3)}{c} \epsilon^{ijk} C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \hat{\psi}^{(3),\alpha=k} \Big] (w) \\
&+ \frac{1}{(z-w)} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-324i(c+2)\hat{J}^i\hat{G}^{i+1}\hat{G}^{i+2} \right. \right. \\
&- 36i(c+6)\hat{J}^i\hat{J}^{i+1}\partial\hat{J}^{i+2} + \frac{36}{c}i(c^2-21c-54)\hat{J}^i\partial\hat{J}^{i+1}\hat{J}^{i+2} \\
&+ 144i(2c+3)\hat{J}^{i+1}\hat{G}^i\hat{G}^{i+2} - 144i(2c+3)\hat{J}^{i+2}\hat{G}^i\hat{G}^{i+1} \\
&+ 72(3c^2+7c+6)\delta^{ij}\hat{T}\hat{T} - 36(c+6)\hat{T}\hat{J}^i\hat{J}^i - 144(2c+3)\delta^{ij} \sum_k \hat{T}\hat{J}^k\hat{J}^k \\
&- 6(8c+9)(c+6)\partial\hat{G}^i\hat{G}^i + 18(3c^2-10c-24)\delta^{ij} \sum_k \partial\hat{G}^k\hat{G}^k \\
&- 18(c^2-7c-6)\delta^{ij} \sum_k \partial\hat{J}^k\partial\hat{J}^k - \frac{18}{c}(c^3+6c^2+48c+72)\partial\hat{J}^i\partial\hat{J}^i \\
&+ \frac{3}{c}(10c^3+48c^2-243c-378)\partial^2\hat{J}^i\hat{J}^i - 3(c-6)(4c+15)\delta^{ij} \sum_k \partial^2\hat{J}^k\hat{J}^k \\
&+ \left. 6(2c+3)(c^2-3c+18)\partial^2\hat{T} \right)_{j=i} \\
&+ \frac{1}{(c+1)(c+6)(2c-3)} \left(6(17c^2+33c+18)\hat{G}^i\partial\hat{G}^{i+1} + 18i(c+6)\hat{J}^i\hat{G}^i\hat{G}^{i+2} \right)
\end{aligned}$$

$$\begin{aligned}
& - 18i(7c+6)\hat{J}^i\hat{J}^i\partial\hat{J}^{i+2} - 9(2c^2-47c-66)\hat{J}^i\partial^2\hat{J}^{i+1} - 18i(c+6)\hat{J}^{i+1}\hat{G}^{i+1}\hat{G}^{i+2} \\
& - 162i(c+2)\hat{J}^{i+1}\hat{J}^{i+1}\partial\hat{J}^{i+2} - 36(c+6)\hat{T}\hat{J}^i\hat{J}^{i+1} + 24i(5c^2+15c+18)\hat{T}\partial\hat{J}^{i+2} \\
& + 18(3c^2-8c-12)\partial\hat{G}^i\hat{G}^{i+1} - \frac{18}{c}i(17c^2+27c-18)\partial\hat{J}^i\hat{J}^i\hat{J}^{i+2} \\
& - \frac{18}{c}(c^3-11c^2-33c-18)\partial\hat{J}^i\partial\hat{J}^{i+1} - \frac{54}{c}i(5c^2+7c+6)\partial\hat{J}^{i+1}\hat{J}^{i+1}\hat{J}^{i+2} \\
& - 216i(2c+3)\partial\hat{J}^{i+2}\hat{J}^{i+2}\hat{J}^{i+2} + 6i(20c^2+57c+54)\partial\hat{T}\hat{J}^{i+2} \\
& + \frac{3}{c}(16c^3+9c^2-99c-54)\partial^2\hat{J}^i\hat{J}^{i+1} + i(2c^3-77c^2-249c-234)\partial^3\hat{J}^{i+2} \Big)_{j=i+1} \\
& - \frac{3}{c}\hat{\phi}^{(2),i}\hat{\phi}^{(2),j} \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{9(3c-1)}{5(c-3)c}\delta^{ij}\hat{G}^k\hat{\phi}^{(\frac{5}{2}),k} - \frac{3(c+3)}{5(c-3)c}\delta^{ij}\hat{J}^k\hat{\psi}^{(3),k} + \frac{18(3c-1)}{5(c-3)c}\delta^{ij}\hat{T}\hat{\psi}^{(2)} \right. \\
& + \frac{3(c-7)}{10(c-3)}\delta^{ij}\partial^2\hat{\psi}^{(2)} + \frac{3}{(c-3)}\hat{G}^i\hat{\phi}^{(\frac{5}{2}),j} + \frac{3}{(c-3)}\hat{G}^j\hat{\phi}^{(\frac{5}{2}),i} \\
& + \frac{3}{(c-3)}\hat{J}^i\hat{\psi}^{(3),j} + \frac{3}{(c-3)}\hat{J}^j\hat{\psi}^{(3),i} + \frac{3i(7c-9)}{5(c-3)c}\epsilon^{ijk}\partial(\hat{J}^k\hat{\psi}^{(2)}) \\
& \left. - \frac{i(c+3)}{10(c-3)}\epsilon^{ijk}\partial\hat{\psi}^{(3),k} \right) + C_{(\frac{3}{2})(\frac{3}{2})}^{(\frac{5}{2})} \left(\frac{6}{5}i\epsilon^{ijk}\partial\hat{\phi}^{(3),k} + \frac{4}{5}\delta^{ij}\hat{\phi}^{(4)} \right) \\
& + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(\frac{1}{2}i\epsilon^{ijk}\partial\hat{\psi}^{(3),\alpha=k} + \frac{1}{2}\hat{\psi}^{(4),i,\alpha=j} + \frac{1}{2}\hat{\psi}^{(4),j,\alpha=i} \right) \Big] (w) + \dots, \\
\hat{\psi}^{(\frac{5}{2}),i}(z)\hat{\phi}^{(3)}(w) & = \frac{1}{(z-w)^4}\frac{12(5c-6)}{(2c-3)}\hat{G}^i(w) \\
& + \frac{1}{(z-w)^3}\left[\frac{1}{(c+1)(2c-3)}\left(12i(2c+3)\epsilon^{ijk}\hat{J}^j\hat{G}^k + 4c(5c+3)\partial\hat{G}^i\right) \right. \\
& - \left. \frac{(c+3)}{2c}C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\hat{\phi}^{(\frac{5}{2}),i}\right](w) \\
& + \frac{1}{(z-w)^2}\left[\frac{1}{(c+1)(c+6)(2c-3)}\left(9(37c+54)\hat{J}^i\hat{J}^j\hat{G}^j - 252(2c+3)\hat{J}^j\hat{J}^j\hat{G}^i \right. \right. \\
& + 6i(11c^2-81c-126)\epsilon^{ijk}\hat{J}^j\partial\hat{G}^k + 6(55c^2+87c+54)\hat{T}\hat{G}^i \\
& - 3i(25c^2-144c-252)\epsilon^{ijk}\partial\hat{J}^j\hat{G}^k \\
& \left. \left. + (5c^3-72c^2-279c-270)\partial^2\hat{G}^i\right) - \frac{9}{2c}\hat{\psi}^{(\frac{3}{2})}\hat{\phi}^{(2),i} \right. \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\left(\frac{3(23c-21)}{10(c-3)c}\hat{G}^i\hat{\psi}^{(2)} + \frac{3i(13c-21)}{10(c-3)c}\epsilon^{ijk}\hat{J}^j\hat{\phi}^{(\frac{5}{2}),k} - \frac{(2c^2+21c-45)}{10(c-3)c}\partial\hat{\phi}^{(\frac{5}{2}),i}\right) \\
& + \left. \frac{7}{5}C_{(\frac{3}{2})(\frac{3}{2})}^{(\frac{5}{2})}\hat{\psi}^{(\frac{7}{2}),i} - \frac{1}{2}iC_{(\frac{3}{2})(\frac{5}{2})}^{(3)}\epsilon^{ijk}\hat{\phi}^{(\frac{7}{2}),j,\alpha=k}\right](w) \\
& + \frac{1}{(z-w)}\left[\frac{1}{(c+1)(c+6)(2c-3)}\left(9(17c+30)\partial\hat{J}^i\hat{J}^j\hat{G}^j + 9(17c+30)\hat{J}^i\hat{J}^j\partial\hat{G}^j \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 9(13c+6)\hat{J}^i\partial\hat{J}^j\hat{G}^j - 216(2c+3)\partial\hat{J}^j\hat{J}^j\hat{G}^i - 108(2c+3)\hat{J}^j\hat{J}^j\partial\hat{G}^i \\
& + 6i(5c^2-33c-54)\epsilon^{ijk}\hat{J}^j\partial^2\hat{G}^k + 54(3c^2+7c+6)\partial\hat{T}\hat{G}^i \\
& + 6(19c^2+3c-18)\hat{T}\partial\hat{G}^i - 21ic(c+6)\epsilon^{ijk}\partial\hat{J}^j\partial\hat{G}^k - 27i(c^2-8c-12)\epsilon^{ijk}\partial^2\hat{J}^j\hat{G}^k \\
& + (c^3-32c^2-75c-54)\partial^3\hat{G}^i \Big) + \frac{3}{2c}\partial\hat{\psi}^{(\frac{3}{2})}\hat{\phi}^{(2),i} - \frac{9}{2c}\hat{\psi}^{(\frac{3}{2})}\partial\hat{\phi}^{(2),i} \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(\frac{9(4c-3)}{10(c-3)c}\hat{G}^i\partial\hat{\psi}^{(2)} - \frac{3i}{2(c-3)}\epsilon^{ijk}\hat{G}^j\hat{\psi}^{(3),k} - \frac{3}{(c-3)}\hat{J}^i\hat{\phi}^{(\frac{7}{2})} \right. \\
& - \frac{3}{(c-3)}\hat{T}\hat{\phi}^{(\frac{5}{2}),i} + \frac{3i(c-12)}{5(c-3)c}\epsilon^{ijk}\partial\hat{J}^j\hat{\phi}^{(\frac{5}{2}),k} + \frac{3(7c-9)}{10(c-3)c}\partial\hat{G}^i\hat{\psi}^{(2)} \\
& - \frac{(c^2+18c-45)}{20c(c-3)}\partial^2\hat{\phi}^{(\frac{5}{2}),i} + \frac{3i(7c-9)}{10(c-3)c}\epsilon^{ijk}\hat{J}^j\partial\hat{\phi}^{(\frac{5}{2}),k} \Big) \\
& + \frac{3}{5}C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}\partial\hat{\psi}^{(\frac{7}{2}),i} + C_{(\frac{3}{2})(\frac{5}{2})}^{(3)} \left(-\frac{1}{4}i\epsilon^{ijk}\partial\hat{\phi}^{(\frac{7}{2}),j,\alpha=k} - \frac{1}{2}\hat{\phi}^{(\frac{9}{2}),\alpha=i} \right) \Big] (w) + \dots, \\
\hat{\phi}^{(3)}(z)\hat{\phi}^{(3)}(w) & = \frac{1}{(z-w)^6}2(5c-3) + \frac{1}{(z-w)^4} \left[\frac{1}{(c+1)(2c-3)} \left(24(5c^2-9)\hat{T} \right. \right. \\
& - 36(c-1)\hat{J}^i\hat{J}^i \Big) + \frac{3(c-7)}{2c}C_{(\frac{3}{2})(\frac{3}{2})}^{(2)}\hat{\psi}^{(2)} \Big] (w) \\
& + \frac{1}{(z-w)^3} \left[\frac{1}{2}\partial(\text{pole four}) \right] (w) \\
& + \frac{1}{(z-w)^2} \left[\frac{1}{(c+1)(c+6)(2c-3)} \left(-144i(2c+3)\epsilon^{ijk}\hat{J}^i\hat{G}^j\hat{G}^k \right. \right. \\
& + 12(7c^2-42c-72)\partial\hat{G}^i\hat{G}^i - \frac{18}{c}(3c^3-8c^2+36c+72)\partial\hat{J}^i\partial\hat{J}^i \\
& + 18(c^2+15c+18)\partial^2\hat{J}^i\hat{J}^i - 288(2c+3)\hat{T}\hat{J}^i\hat{J}^i \\
& + 18(c^3-4c^2+18c+36)\partial^2\hat{T} + 24(16c^2+27c+18)\hat{T}\hat{T} \Big) - \frac{18}{c}\partial\hat{\psi}^{(\frac{3}{2})}\hat{\psi}^{(\frac{3}{2})} \\
& + C_{(\frac{3}{2})(\frac{3}{2})}^{(2)} \left(-\frac{3(11c-12)}{10(c-3)c}\hat{G}^i\hat{\phi}^{(\frac{5}{2}),i} + \frac{9(c-2)}{5(c-3)c}\hat{J}^i\hat{\psi}^{(3),i} + \frac{9(c-5)(c+3)}{40(c-3)c}\partial^2\hat{\psi}^{(2)} \right. \\
& + \frac{12(4c-3)}{5(c-3)c}\hat{T}\hat{\psi}^{(2)} \Big) + \frac{8}{5}C_{(\frac{3}{2})(3)}^{(\frac{5}{2})}\hat{\phi}^{(4)} + \frac{1}{4}C_{(\frac{3}{2})(\frac{5}{2})}^{(3)}\hat{\psi}^{(4),i,\alpha=i} \Big] (w) \\
& + \frac{1}{(z-w)} \left[-\frac{1}{24}\partial^3(\text{pole four}) + \frac{1}{2}\partial(\text{pole two}) \right] (w) + \dots. \tag{G.3} \{?\}
\end{aligned}$$

There are also the nonlinear terms between the higher spin currents in (G.3). Compared to the previous OPEs in this Appendix, the coefficients appearing in the higher spin currents in (G.3) are not simply equal to those in Appendix F. This implies that the extra terms appearing on the right hand sides of (8.4) can contribute to the higher spin current dependent terms in (G.3). The presentation in the first-order pole of the OPE between the higher spin- $\frac{5}{2}$ currents

is rather complicated and we can analyze the notations here by doing similar procedures in (G.2). The cases $(i, j) = (2, 2)$ and $(3, 3)$ can be read off from the case $(i, j) = (1, 1)$ while the cases $(i, j) = (2, 3)$ and $(3, 1)$ can be obtained from the case $(i, j) = (1, 2)$.

H Further $\mathcal{N} = 3$ description for low (N, M) cases

In this Appendix, we describe the $\mathcal{N} = 3$ OPEs for different (N, M) cases which are not explained in the main text. There is no higher spin-1 current for all of the following cases as before and it is identically and trivially zero. Then we construct the nontrivial lowest higher spin- $\frac{3}{2}$ current for each case.

H.1 The $(N, M) = (2, 1)$ case

Since $M = 1$, there are only $SU(N)$ adjoint fermions. One can check that there exists the higher spin- $\frac{3}{2}$ current and it is similar to the one for $(N, M) = (2, 2)$ case by removing the $SU(M)$ adjoint fermions. From the general expression (3.11), one has

$$\psi^{(\frac{3}{2})}(z) \sim \left(a J_1^\alpha \Psi^\alpha + J_2^\alpha \Psi^\alpha + j^\alpha \Psi^\alpha \right)(z), \quad (\text{H.1}) \{?\}$$

where a is some constant $a(2, 1)$ defined in Appendix (D.1) and the index α is $SU(2)$ adjoint index. The spin- $\frac{1}{2}$ and spin-1 currents are given in Appendix C. Then we can calculate the various OPEs based on the higher spin- $\frac{3}{2}$ current (H.1) and it turns out that there are no other higher spin currents in the OPE $\Phi^{(\frac{3}{2})}(Z_1) \Phi^{(\frac{3}{2})}(Z_2)$ and can be summarized as follows in the simplified notation

$$\left[\Phi^{(\frac{3}{2})} \right] \cdot \left[\Phi^{(\frac{3}{2})} \right] = [\mathbf{I}]. \quad (\text{H.2}) \{?\}$$

Here $[\mathbf{I}]$ stands for the $\mathcal{N} = 3$ superconformal family of the identity operator. The explicit result can be seen from (7.2) by neglecting all the higher spin currents appearing in the right hand side. Furthermore, we can try to find whether the higher spin-2 current exists or not. If we require that the general higher spin-2 ansatz should satisfy the primary conditions given in Appendix B, this higher spin-2 current vanishes identically. This argument also holds for the other higher spin- $\frac{5}{2}$ current.

H.2 The $(N, M) = (3, 1), (4, 1), (5, 1)$ cases

What happens for other N values for fixed $M = 1$? Let us increase the N value for fixed $M = 1$. As in the above case, the higher spin- $\frac{3}{2}$ current is given by

$$\psi^{(\frac{3}{2})}(z) \sim \left(aJ_1^\alpha \Psi^\alpha + J_2^\alpha \Psi^\alpha + j^\alpha \Psi^\alpha \right)(z), \quad (\text{H.3}) \{?\}$$

which comes from (3.11) or (H.1). Again, a is some constant $a(N, 1)$ in Appendix (D.1) and the index α is $SU(N = 3, 4, 5)$ adjoint index. For $M = 1$, there are no $SU(M)$ adjoint fermions in (H.3). Again, there are no higher spin-2 currents for each case in general. However, there exists the higher spin- $\frac{5}{2}$ current for each case. Moreover, the $\mathcal{N} = 3$ OPE between the lowest higher spin- $\frac{3}{2}$ current can be summarized by

$$\left[\Phi^{(\frac{3}{2})} \right] \cdot \left[\Phi^{(\frac{3}{2})} \right] = [\mathbf{I}] + \left[\Phi^{(\frac{5}{2})} \right], \quad (\text{H.4}) \{?\}$$

from (H.3). The explicit result is the same as (7.2) once we ignore the higher spin currents in the right hand side of (7.2) except $\Phi^{(\frac{5}{2})}(Z_2)$.

When $M = 1$, our coset is (that is, we can ignore the $\hat{S}U(M)_{2N+M}$ factor in the denominator of (1.1)) given by

$$\frac{\hat{S}U(N+M)_{N+M} \oplus \hat{S}O(2NM)_1}{\hat{S}U(N)_{N+2M} \oplus \hat{U}(1)_{2NM(N+M)^2}}. \quad (\text{H.5}) \{?\}$$

When the free fermions, $\Psi^\rho(z)$ and $\Psi^{u(1)}(z)$, are decoupled, the above coset (H.5) is dual to the following coset (see (4.13) in [18])

$$\frac{\hat{S}U(N+2M)_N}{\hat{S}U(N)_N \oplus \hat{U}(1)_{2N^2M(N+2M)}}. \quad (\text{H.6}) \{?\}$$

In the large N limit, the vacuum character of (H.6) is (see (3.13) with replaced M by $2M$ in [18])

$$\chi_0^{2M} = \left(\prod_{s=2}^{\infty} z_B^{(s)}(q) z_F^{(s)}(q) \right)^{4M^2} \left(z_B^{(1)}(q) \right)^{4M^2-1}, \quad (\text{H.7}) \{?\}$$

where $z_B^{(s)}(q) \equiv \prod_{n=s}^{\infty} \frac{1}{(1-q^n)}$ and $z_F^{(s)}(q) \equiv \prod_{n=s}^{\infty} (1+q^{n-\frac{1}{2}})$ are defined in (3.7) and (3.13) of [18]. When $M = 1$, the vacuum character (H.7) implies that there are three spin-1 currents and four spin- s currents for every half-integer spin s greater than 1. This interpretation partly matches with our spin content in the lower spin cases when we ignore spin- $\frac{1}{2}$ current $\Psi(z)(= \sqrt{\frac{NM}{2}} \Psi^{u(1)}(z))$ in the $\mathcal{N} = 3$ superconformal algebra (8.3). Note that there are no

$\hat{SU}(M)$ currents $\Psi^\rho(z)$ because of $M = 1$. The previous $(N, M) = (2, 1)$ case matches with (H.7) for spin $s = 1, \frac{3}{2}$, and 2. Recall that the $\mathcal{N} = 3$ superconformal algebra contains three spin-1, three spin- $\frac{3}{2}$ and one spin-2 currents while the higher spin- $\frac{3}{2}$ current $\Phi^{(\frac{3}{2})}(Z)$ contains the one higher spin- $\frac{3}{2}$ and three higher spin-2 currents.

The present case matches with (H.7) for spin $s = 1, \frac{3}{2}, 2, \frac{5}{2}$, and 3. In this case, there are three higher spin- $\frac{5}{2}$ currents from the lowest higher spin- $\frac{3}{2}$ current $\Phi^{(\frac{3}{2})}(Z)$ and one higher spin- $\frac{5}{2}$ current from $\Phi^{(\frac{5}{2})}(Z)$. Moreover, one sees that one higher spin-3 current from $\Phi^{(\frac{3}{2})}(Z)$ and three higher spin-3 currents from $\Phi^{(\frac{5}{2})}(Z)$. We expect that if we increase the N values and find more higher spin currents, then the spin content will match further beyond the higher spin-3 current.

H.3 The $(N, M) = (2, 2)$ case

What happens for different M value? Let us increase the M value. For $M = 2$, the nontrivial case arises for $N = 2$. From the general expression (3.11), one has

$$\psi^{(\frac{3}{2})}(z) \sim d\left(aJ_1^\alpha\Psi^\alpha + J_2^\alpha\Psi^\alpha + j^\alpha\Psi^\alpha\right)(z) + \left(bJ_1^\rho\Psi^\rho + J_2^\rho\Psi^\rho + j^\rho\Psi^\rho\right)(z). \quad (\text{H.8}) \{?\}$$

Note that the index ρ runs over $1, 2, \dots, M^2 - 1$ for general M . For $M \neq 1$, we have the terms having $SU(M)$ adjoint ρ indices in (H.8). There are new higher spin currents $\Phi^{(2)}(Z)$, $\Phi^{(\frac{5}{2})}(Z)$, and $\Phi^{(3),i}(Z)$ which are not present for $M = 1$ cases described before¹⁰. Compared to the previous cases (H.2) and (H.4), we have the following OPE

$$\left[\Phi^{(\frac{3}{2})}\right] \cdot \left[\Phi^{(\frac{3}{2})}\right] = [\mathbf{I}] + \left[\Phi^{(2)}\right] + \left[\Phi^{(\frac{5}{2})}\right] + \theta^{3-i} \left[\Phi^{(3),i}\right], \quad (\text{H.9}) \{?\}$$

which appeared in section 7. So far, the vacuum character in this case corresponding to (H.7) is not known. It would be interesting to obtain this vacuum character by dividing the $\hat{SU}(M)$ factor from the result in [36].

H.4 The $(N, M) = (3, 2)$ case

Let us increase the N value for fixed $M = 2$. Once again, the higher spin- $\frac{3}{2}$ current is given by (3.11) for $(N, M) = (3, 2)$

$$\psi^{(\frac{3}{2})}(z) \sim d\left(aJ_1^\alpha\Psi^\alpha + J_2^\alpha\Psi^\alpha + j^\alpha\Psi^\alpha\right)(z) + \left(bJ_1^\rho\Psi^\rho + J_2^\rho\Psi^\rho + j^\rho\Psi^\rho\right)(z). \quad (\text{H.10}) \{?\}$$

¹⁰We can also construct the new higher spin currents $\Phi^{(\frac{3}{2}),i}(Z)$ for $(N, M) = (2, 2)$ case. However, they do not appear in the OPE $\Phi^{(\frac{3}{2})}(Z_1)\Phi^{(\frac{3}{2})}(Z_2)$.

In general, the new higher spin fields can arise in the coset when (N, M) increase. We can investigate whether new higher spin fields arise in the OPE $\Phi^{(\frac{3}{2})}(Z_1)\Phi^{(\frac{3}{2})}(Z_2)$ or not. It is sufficient to see the singular terms of the OPEs between the lowest component field $\psi^{(\frac{3}{2})}(z)$ and all the other component fields of $\Phi^{(\frac{3}{2})}(Z)$ with (H.10) using the $\mathcal{N} = 3$ supersymmetry. It turns out that there are no new higher spin currents. Again, we summarize the OPE as follows as in (H.9):

$$\left[\Phi^{(\frac{3}{2})}\right] \cdot \left[\Phi^{(\frac{3}{2})}\right] = [\mathbf{I}] + [\Phi^{(2)}] + [\Phi^{(\frac{5}{2})}] + \theta^{3-i} [\Phi^{(3),i}]. \quad (\text{H.11}) \{?\}$$

The explicit check for the other values of (N, M) is computationally involved. If there are extra higher spin currents in the above (H.11) for large (N, M) values, we expect that they (and their descendant fields) will appear linearly [1].

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